Low Gain Controller Design with Regional Pole Placement Constraints

Thesis submitted in partial fulfillment of the requirements for the degree of

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in

Electrical Engineering

(Specialization: Control & Automation)

by

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Certificate

This is to certify that the work in the thesis entitled "***Low Gain Controller Design with Regional Pole Placement Constraints***" by ***Usha Mahato*** is a record of an original research work carried out by her under my supervision and guidance in partial fulfillment of the requirements for the award of the degree of Master of Technology with the specialization of **Control** & **Automation** in the department of **Electrical Engineering**, National Institute of Technology Rourkela. Neither this thesis nor any part of it has been submitted for any degree or academic award elsewhere.

Place: NIT Rourkela
Date: 21 May 2013

Prof. Sandip Ghosh
Professor, EE Department
NIT Rourkela, Odisha

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Last, but not the least, I would like to dedicate this thesis to my family, for their love, support, patience, and understanding.

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Abstract

An iterative linear matrix inequalities (ILMIs) algorithm is presented for centralized and decentralized state feedback controller designs. The controller is designed in such a way that places the closed loop poles under desired area and bounds near the boundary region with low gain controller. The application of algorithm is demonstrated through simulation studies of two-area power system model and formation control of unmanned aerial vehicles.

Contents

List of Figures

Symbols & Abbreviations

$\in$	:	Belongs to
$\mathbb{R}$	:	The set of real numbers
$\mathbb{R}^n$	:	The set of real n vectors
$\mathbb{R}^{n\times m}$	:	The set of real $n \times m$ vectors
$\|.\|$	:	Euclidean norm
$X > 0$	:	Positive definite matrix
$X \geq 0$	:	Positive semi-definite matrix
LFC	:	Load Frequency Control
LMI	:	Linear Matrix Inequality
BMI	:	Bilinear Matrix Inequality
UAV	:	Unmanned Aerial vehicle

Chapter 1

Review on Decentralized Control and Pole Placement method: LMI based design

Chapter 1

Review on Decentralized Control and Pole Placement method: LMI based design

1.1 Introduction

The basic concepts presented in this chapter are required for understanding formulation of control problem for any system and finding its solution by using a standard method.

This chapter is organized as follows: section 1.2 this section gives a brief review on literature. Section 1.3 gives the concept of decentralized control strategies. Section 1.4 reviews inclusion principle which provide mathematical framework for the expansion/contraction of large scale system with overlapped subsystems. Section 1.5 this section presents a review on convex optimization and linear matrix inequality(LMI) based controller design theories, which are very essential to understand the basic concept behind presented optimization problem. Based on LMI different type of optimization problem can be formulated such as $H\infty$ optimization problem, decentralized control structure depending on given problem scenarios, optimal system realization and robust stability etc. Section 1.6 discusses LMI based pole placement method, which helps to place the closed loop poles under some LMI regions. Section 1.7 in this section state feedback control law is applied for a system with pole placement and minimum gain constraint.Section 1.8 gives a brief introduction of homotopy method for solving decentralized control problem.

Section 1.9 gives a brief review on considered application problems. Section 1.10 this section contains outline of thesis.

1.2 Literature Review

Decentralized control approach has been become the most popular and preferred control strategy for large scale system for over many past years [1,2]. The overview for analysis and solving methods for design problems can be found in [3]. In the decentralized control the whole system is considered as interconnection of subsystems. A local controller is designed for individual one based on available local information, which ensures stability and performance depending on one requirement — the local objective. The global objective is the designed local controllers must ensure the stability and the performance of the whole system.

The choice of subsystems affects the performance of control system, which is limited by information structure constraints [4,5]. Based on information structure constraints, decomposition of large scale system is the fundamental pre-requisite step for control designing for breaking a large dimensional system into smaller subsystems [6].

Control design for a system with overlapping subsystems is started by expanded the system into large dimensional, where the subsystems are appeared as disjoint [5,6,8]. The expanded space contains all the necessary information of the original system such that a control law is designed for each subsystem, then contracted back for implementation of control law into original system. For the expansion and contraction of the system, the mathematical framework is known as inclusion principle [4,6].

A good damping response and fast decay rate can be imposed on the system by restricting the closed loop eigen values under the region of intersection of conic sector and a shifted half plane in the complex s-plane [11]. Such a restriction implies that all closed loop poles lie under D-region. This is also known as D-

stability of the system and the technique is known as regional pole placement. By satisfying regional pole placement constraints, a controller is able to guarantee satisfactory transient performance. The regional pole placement with the other design constraints is considered in [9, 10, 12, 13].

Recently, LMIs have become a powerful tool for solving numerous control problems. The LMIs are convex optimization problem, which can be solved efficiently [14, 15]. Control problems such as Lyapunov stability, Reccati inequality etc. can be easily written as LMIs and also multiple LMIs can be written as single LMI with larger dimension. Thus LMIs help for solving a variety of optimization and control problems. Generally the state feedback control problem is expressed as bilinear matrix inequalities (BMIs) optimization problem [16]. One approach for solving this BMIs problem is to convert it into LMIs problem with addition constraints [16]. Another approach is to solve LMIs problem iteratively.

Homotopy approach for solving BMIs optimization problem is one example of second type approach. A path-following method for solving BMIs problem is considered in [17], where BMIs problem is solved by introducing first order approximation into the control variables. This results LMIs optimization problem by neglecting bilinear term which is assumed as a small quantity. Then the resulted LMIs problem is solved for perturbation term that slightly improves the performance of controller. The whole process is repeated until an optimum solution for controller is achieved. In homotopy method [18] for solving decentralized overlapping control problem based on two homotopy path-followings. Along first path the centralized controller is deformed into decentralized controller in each step. Along second path the decentralized control design problem is linearized and solved.

Conventional methods for load frequency control (LFC) of interconnected power system are PI and PID control, which has wide application in industries. However, their controller parameters are determined by trial and error methods. The centralized approach for LFC based on optimal control theory is considered in [20,21], where control problem is formulated as a cost function and minimization of cost function results an optimal controller.

Recently, decentralized load frequency control of interconnected power system has been become one of the most important research issue. Decentralized control with addition constraints are considered in [22, 23], where the control problem is considered as a convex optimization problem and solved by using standard tool.

Research on formation control of a group of unmanned aerial vehicles has been drawn more attention in the recent years. Centralized control strategies are able to give a global optimum solution if no. of vehicles is less. However, increase in no. of vehicles and constraints preventing their practical implementation. Thus centralized solution of formation control rarely exists. Decentralized control strategies only require local information. Such an approach has advantage such as rapid reconfiguration in the event of single point failure, low cost, reliability etc. Decentralized overlapping control of a group of UAVs is considered in [25].

1.3 Decentralized Controller

While dealing with control problems three steps: modeling, describing qualitative properties and controlling system behaviors are applied. This concept is applicable for centralized control, where a single controller is designed based on whole system information. But centralized control is not reliable and economical for the implementation into large scale system and also increases complexity in the design process. Because there is possibility of losing local data, presence of time delays due to long distance information transfer and presence of uncertainty in the model. Thus, the control problem becomes too large to be controlled and too complex to be solved.

Whereas decentralized approach [2, 7] provides a way to deal with above difficulties by breaking the original system into a no. of subsystem. Each subsystem is controlled by a local controller, which requires a part of global information. Thus decentralized control design solves difficulties encountered in analyzing, designing and implementing control strategies and algorithms in centralized case.

1.4 Inclusion Principle

A large and complex system with overlapped subsystems can be expanded to a space in which subsystems appear as disjoint. In the expanded space a control law based on available information, is designed for each subsystem by using any standard method and then transformed it into a final control law which is implementable into the original system.

Consider two linear time invariant system

S: $\dot{x} = Ax + Bu, \quad x(t_0) = x_0$
where $x \in \mathbb{R}^n$ and $u \in \mathbb{R}^m$ are the state and the control input of original system respectively.

$\tilde{\mathbf{S}}$: $\dot{\tilde{x}} = \tilde{A}\tilde{x} + \tilde{B}\tilde{u}, \quad \tilde{x}(t_0) = \tilde{x}_0$
where $\tilde{x} \in \mathbb{R}^{\tilde{n}}$ and $\tilde{u} \in \mathbb{R}^{\tilde{m}}$ are the state and the control input of expanded system respectively.

Here consider $\tilde{n} > n$ and $\tilde{m} > m$. We can relate the original system S with the expanded system $\tilde{S}$ by the following expansion/contraction matrices

$$V \in \mathbb{R}^{\tilde{n}\times n}, \quad V \in \mathbb{R}^{n\times \tilde{n}}, \quad UV = I \in \mathbb{R}^{n\times n}$$

and

$$R \in \mathbb{R}^{\tilde{m}\times m}, \quad Q \in \mathbb{R}^{m\times \tilde{m}}, \quad QR = I \in \mathbb{R}^{m\times m} \tag{1.1}$$

Definition 1.1: (Inclusion Principle) A system $\tilde{S}$ includes system S, denoted by $\tilde{S} \supset S$, If there exists pairs of matrices (U, V) and (Q, R) such that $UV = I$

and $QR = I$, and for any initial state x_0 and control input u, we have

$$x(t; x_0, u) = U\tilde{x}(t; \tilde{x_0}, \tilde{u}) \tag{1.2}$$

where $\tilde{x} = Vx, x = U\tilde{x}$ and $\tilde{u} = Ru, u = Q\tilde{u}$.

Theorem 1: If $\tilde{S}$ is the expanded form of S then the following are true

$$\tilde{A}V = VA$$

$$\tilde{B}R = VB \tag{1.3}$$

The pictorial representation of inclusion principle is shown in fig.(1.1).

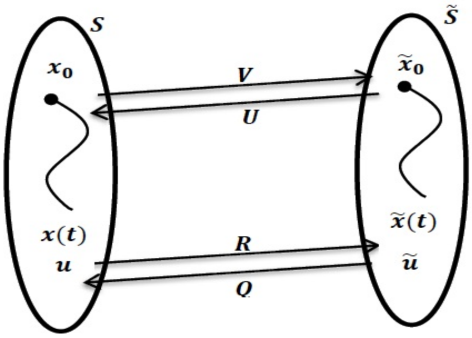


Figure 1.1: Inclusion principle

If static state feedback law is applied for both systems in the following form

$$u = Kx, \qquad K \in \mathbb{R}^{m\times n}$$

$$\tilde{u} = \tilde{K}\tilde{x}, \qquad \tilde{K} \in \mathbb{R}^{\tilde{m}\times\tilde{n}} \tag{1.4}$$

The closed loop system in the original space

$$\mathbf{S:}\ \ \dot{x} = (A + BK)x$$

and in the expanded space

$$\mathbf{\tilde{S}:}\ \ \dot{\tilde{x}} = (\tilde{A} + \tilde{B}\tilde{K})\tilde{x} \tag{1.5}$$

Theorem 2: If $\tilde{S}$ is the expanded form of S then the following are true

$$\tilde{A}V = VA,$$

$$\tilde{B}R = VB$$

and

$$\tilde{K}V = RK \tag{1.6}$$

1.5 Convex Optimization

Definition 1.2: A set C is convex if the line segment between any two points in C lies in C and the following holds:

$$\lambda x_1 + (1-\lambda)\, x_2 \in C \tag{1.7}$$

for any $x_1, x_2 \in C$ and λ with $0 \leq \lambda \leq 1$.

Definition 1.3: A function $f : R^n \to R$ is convex if $dom f$ is a convex set and the following holds:

$$f\left(\lambda x_1 + (1-\lambda)\, x_2\right) \leq \lambda f\left(x_1\right) + (1-\lambda)\, f\left(x_2\right) \tag{1.8}$$

for any $x_1, x_2 \in dom f$ and λ with $0 \leq \lambda \leq 1$.

Geometrically, this inequality means that line segment between $(x, f(x))$ and $(y, f(y))$ lies above the graph of f . f is a concave function if it replaced by $-f$.An affine function holds the above equality, so all affine function are both convex and concave.

Definition 1.4: The optimization problem of a convex function $f : R^n \to R$to be minimized over optimization variable x subject to inequality constraints on convex function of x and equality constraints on affine function of x is a convex optimization problem, i.e.

$$
\begin{aligned}
&\text{minimize } f(x) \\
&\text{subject to } g_i(x) \le 0, \quad (i = 0, 1, ...m) \\
&h_j(x) = 0, \quad (j = 0, 1, ...p)
\end{aligned} \tag{1.9}
$$

where equality constraints is replaced by a pair of inequality constraints $h_j \le 0$ and $h_j \ge 0$.

1.6 Pole Placement in LMI regions

Stability is minimum requirement of any control system. But a good controller should also deliver a sufficient fast and well damped transient response which can be easily achieved by placing the closed loop poles under desired region D as shown in fig.(1.2). Settling time and overshoot depend on the selection of damping ratio $cos\theta$ and speed of the system depends on γ_0 .

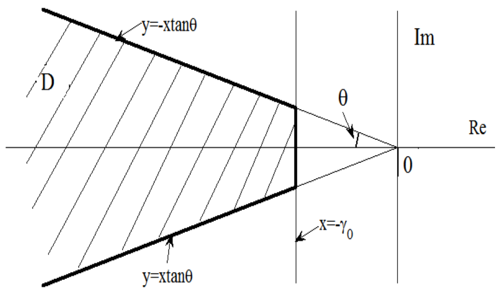


Figure 1.2: Desired Region in the Complex Plane

Definition 1.5:A subset D of the complex plane is called an LMI region if there exist a symmetric matrix L and a matrix M such that

$$D = \Big\{ z \in \mathbb{C} : f_d(z) < 0$$

Where $f_d(z) = L + zM + \bar{z}M^T$ and $f_d(z)$ is called the characteristic function of D. A few examples of LMI regions are

- Half plane $Re(z) = -\alpha$, $f_d(z) = z + \bar{z} + 2\alpha < 0$

- Disk centered at $(-q, 0)$ with radius r,

$$f_d(z) = \begin{bmatrix} -r & q+z \\ z+\bar{z} & -r \end{bmatrix} < 0$$

- Conic sector 2θ,

$$f_d(z) = \begin{bmatrix} sin\theta(z+\bar{z}) & cos\theta(z-\bar{z}) \\ cos\theta(\bar{z}-z) & sin\theta(z+\bar{z}) \end{bmatrix} < 0$$

Note:

- Intersection of LMI regions are LMI regions.
- Any convex region that is symmetric with respect to real axis can be approximated by LMI region to any desired accuracy.
- A real matrix A is D-stable i.e. has all its eigenvalues in the LMI region D if and only if there exists a positive systematic matrix P such that

$$M_D(A, P) = L \bigotimes I + M \bigotimes (PA) + M^T \bigotimes (A^T P) < 0$$

  In which $\bigotimes$ represent Kronecker product [24] of two matrices.

Pole clustering in LMI regions can be considered as LMI optimization problem and is therefore tractable [9, 10, 12, 13] . It is also possible to combine such pole clustering specifications with other design objectives.

1.7 State Feedback Controller Design through Pole Placement

At the starting of the designing process, we select a combination of LMI regions inside which we wish to place the poles of closed loop system:

$$D = \left\{ z \in \mathbb{C} : \begin{array}{c} x < -\gamma_0 \\ \pm x tan\theta < y \end{array} \right. \tag{1.10}$$

where $x = Re(z)$ and $y = Im(z)$ or $x = \frac{z+\bar{z}}{2}$ and $y = \frac{z-\bar{z}}{2j}$

In other words region$-D$ is the intersection of $f_{d1}(z) < 0$ and $f_{d2}(z) < 0$

$$D = \left\{ z \in \mathbb{C} : \begin{array}{c} f_{d1}(z) < 0 \\ f_{d2}(z) < 0 \end{array} \right. \tag{1.11}$$

where

$$f_{d1}(z) = z + \bar{z} + 2\gamma_0$$
$$f_{d2}(z) = \begin{bmatrix} sin\theta(z+\bar{z}) & cos\theta(z-\bar{z}) \\ cos\theta(\bar{z}-z) & sin\theta(z+\bar{z}) \end{bmatrix}$$

The poles of the closed loop system lie inside the area shown in fig.(1.2) if and only if there exist a positive and symmetric matrix P in such a way that it satisfy the following conditions

$$P > 0$$
$$(A+BK)P + P(A+BK)^T + 2\gamma_0 P < 0$$
$$\begin{bmatrix} sin\theta[(A+BK)P + P(A+BK)^T] & cos\theta[(A+BK)P - P(A+BK)^T] \\ cos\theta[(A+BK)P - P(A+BK)^T]^T & sin\theta[(A+BK)P + P(A+BK)^T] \end{bmatrix} < 0 \tag{1.12}$$

Which is a bilinear matrix inequalities optimization problem (non-convex). One approach to solve the above problem is to convert it into convex problem by taking addition constraints. Consider $Y = KP$, then Eq.(1.12) becomes

$$P > 0$$
$$AP + PA^T + BY + Y^T B^T + 2\gamma_0 P < 0$$
$$\begin{bmatrix} sin\theta[AP + PA^T + BY + Y^T B^T] & cos\theta[AP - PA^T + BY - Y^T B^T] \\ cos\theta[AP - PA^T + BY - Y^T B^T]^T & sin\theta[AP + PA^T + BY + Y^T B^T] \end{bmatrix} < 0 \tag{1.13}$$

There are 2 unknown P and Y, which are obtained by solving the above (1.13) LMIs optimization problem. The control gain is given by

$$K = YP^{-1} \tag{1.14}$$

Constraints for Gain Optimization

The control gain may be high for this reason we need to consider the following constraints

$$Y^T Y < k_y I \tag{1.15}$$
$$P^{-1} < k_p I \tag{1.16}$$

where k_y and k_p are positive number.By appling *Schur* Complement

$$\begin{bmatrix} -k_y I & Y^T \\ Y & -I \end{bmatrix} < 0,$$
$$\begin{bmatrix} P & I \\ I & k_p I \end{bmatrix} > 0$$

1.8 Homotopy method for decentralized control

Homotopy method is an iterative algorithm to solve the bilinear matrix inequality (BMI) problem for decentralized state feedback control. It follows two homotopy paths: along the first path the full centralized controller is iteratively deformed into decentralized controller. Along the second path the decentralized control design problem (i.e. BMI problem) is locally linearized and solved.

The BMI problem is linearized by using a first order perturbation approximation. Then the resulting LMI problem is solved to compute the perturbation term which slightly improves the performance and has value small enough so that perturbed variables satisfy the initial BMI problem. The centralized solution of control problem is taken as initial solution for starting iterative algorithm.

1.9 Application Problems

1.9.1 Load Frequency Control

In an interconnected power system power demand changes according to end users, this directly affects frequency and tie line power flow. The objectives of load frequency control (LFC) are to minimize the deviations in frequency and tie line power flow and to maintain steady state errors zero.

1.9.2 Formation Control

In formation control [25], a group of unmanned aerial vehicles move in a specified pattern, where may exist one or more leader and other followers. Different control strategies can be adopted depending on specific information structure constraints, to control the whole system.

1.10 Thesis Organization

The rest of the thesis is organized as follows:

- **Chapter 2:** Design algorithms for both non-iterative and iterative case are presented.
- **Chapter 3:** Load frequency control of power system is presented, where a model is developed for ith area and centralized and decentralized control strategies are presented with simulation results by using design algorithms (presented in chapter 2).
- **Chapter 4:** Formation control of a group of unmanned aerial vehicles is presented with simulation results, where the formation is modeled as a sys-

tem with interconnected subsystems. A decentralized overlapping controller is designed based on inclusion principle and pole placement and solved by using design algorithms (presented in chapter 2).

- **Chapter 5:** This chapter presents discussions and conclusions on results.

Chapter 2

Design Algorithms

Chapter 2

Design Algorithms

This chapter presents design algorithms for both non-iterative and iterative case. And also presents design algorithm of homotopy method for solving decentralized control problem.

In non-iterative algorithm the BMI problem is solved by converting it into convex optimization problem and taking additional constraints. For iterative algorithm the BMI problem is solved iteratively. In first step BMI problem is solved by converting it into convex optimization problem and taking additional constraints. Solution of first step is taken as initial solution. In the second and third step BMI problem is solved by fixing one matrix variable and solving resulted LMI problem for other variable. In fourth step process is repeated until it results a low gain controller with required accuracy. Minimization of controller gain is achieved by taking the following constraints on P and K.

$$P < \beta I$$

and

$$K^T K = \alpha I$$

By appling *Schur* Complement

$$\begin{bmatrix} \alpha I & K^T \\ K & I \end{bmatrix} > 0$$

Now objective becomes to minimize β in second step and α in next step. According to the requirement P may be solved with constraints or may not be.

2.1 Non-iterative Algorithm

The following optimization problem is solved for $P > 0$,which gives controller gain K:

Subject to $P > 0$

$$AP + PA^T + BY + Y^T B^T + 2\gamma_0 P < 0$$

$$\begin{bmatrix} sin\theta[AP + PA^T + BY + Y^T B^T] & cos\theta[AP - PA^T + BY - Y^T B^T] \\ cos\theta[AP - PA^T + BY - Y^T B^T]^T & sin\theta[AP + PA^T + BY + Y^T B^T] \end{bmatrix} < 0$$

Here K is simply obtained by $K = YP^{-1}$.

2.2 Iterative Algorithm

To develop iterative algorithm we use D-K type iteration algorithm. The following steps are followed one by one for calculation of optimal controller gain of the system:

1. Initialize iteration number $i = 0$. Solve for $P > 0$ the following optimization problem :

   Minimize $a_1 k_y + a_2 k_p$

   Subject to $P > 0$

   $$AP + PA^T + BY + Y^T B^T + 2\gamma_0 P < 0$$

   $$\begin{bmatrix} sin\theta[AP + PA^T + BY + Y^T B^T] & cos\theta[AP - PA^T + BY - Y^T B^T] \\ cos\theta[AP - PA^T + BY - Y^T B^T]^T & sin\theta[AP + PA^T + BY + Y^T B^T] \end{bmatrix} < 0$$

   $$\begin{bmatrix} -k_y I & Y^T \\ Y & -I \end{bmatrix} < 0$$

   $$\begin{bmatrix} P & I \\ I & k_p I \end{bmatrix} > 0$$

   If it does not give a feasible solution then abandon the process , the algorithm does not exist a feasible solution. If it exists then intialize $K_{(i)} =$

YP^{-1} and $\alpha_{(i)} = \left\| K_{(i)}^T K_{(i)} \right\|$. a_1 and a_2 represent positive weights.

2. Increase the iteration number by $i = i + 1$ and assign $K_{(i-1)} = YP^{-1}$. Find a feasible solution for $P > 0$ by solving the following optimization problem for given $K_{(i-1)}$ obtained from step1 :

   Minimize β

   Subject to $P > 0$

   $P < \beta I$

   $AP + BKP + PA^T + PK^T B^T + 2\gamma_0 P < 0$

   $$\begin{bmatrix} sin\theta[AP + BKP + PA^T + PK^T B^T] & cos\theta[AP + BKP - PA - PK^T B^T] \\ cos\theta[AP + BKP - PA - PK^T B^T]^T & sin\theta[AP + BKP + PA^T + PK^T B^T] \end{bmatrix} < 0$$

3. Solve the following optimization problem for given P obtained from step2:

   Minimize α

   $AP + BKP + PA^T + PK^T B^T + 2\gamma_0 P < 0$

   $$\begin{bmatrix} sin\theta[AP + BKP + PA^T + PK^T B^T] & cos\theta[AP + BKP - PA - PK^T B^T] \\ cos\theta[AP + BKP - PA - PK^T B^T]^T & sin\theta[AP + BKP + PA^T + PK^T B^T] \end{bmatrix} < 0$$

   $$\begin{bmatrix} \alpha I & K^T \\ K & I \end{bmatrix} > 0$$

4. Check whether the $\frac{\left|\alpha_{(i-1)} - \alpha_{(i)}\right|}{\alpha_{(i)}} < \varepsilon$ or not, where ε is a specified positive small quantity. If it satisfies then stop and the obtained control gain $K = K_{(i)}$. Otherwise go to Step2 .

2.3 Method for decentralized overlapping control

While solving formation control problem step 1 does not give feasible solution for individual subsystem. So we choose homotopy method [18].

In centralized control of formation control of unmanned aerial vehicle, there exists a symmetric matrix P which satisfy the following inequalies

$$
\begin{aligned}
&P > 0 \\
&P(A+BK) + (A+BK)^T P + 2\gamma_0 I < 0 \\
&\begin{bmatrix} sin\theta[P(A+BK) + (A+BK)^T P] & cos\theta[P(A+BK) - (A+BK)^T P] \\ cos\theta[P(A+BK) - (A+BK)^T P]^T & sin\theta[P(A+BK) + (A+BK)^T P] \end{bmatrix} < 0
\end{aligned}
\tag{2.1}
$$

It is cleared that the above inequities is a BMIs problem which give a feasible solution by converting Eq.(2.1) into convex problem in case of K having no specific structure. However, it does not give solution if K has decentralized structure K_D. To overcome this problem homotopy method for decentralized control is going to be discussed.

2.3.1 Double Homotopy Method

Suppose P_0 and K_0 denote the centralized solution of the system. Now let us introduce small perturbation ΔP and ΔK into matrix variables P and K respectively. Then

$$P = P_0 + \Delta P$$

$$K = K_0 + \Delta K$$

$$
\begin{aligned}
P(A+BK) &= (P_0 + \Delta P)[A + B(K_0 + \Delta K)] \\
&= P_0 A + P_0 B K_0 + P_0 B \Delta K + \Delta P A + \Delta P B K_0 + \Delta P B \Delta K \\
&\simeq P_0(A + BK_0) + \Delta P(A + BK_0) + P_0 B \Delta K
\end{aligned}
$$

and

$$
\begin{aligned}
(A+BK)^T P &= [A+B(K_0+\Delta K)]^T (P_0+\Delta P) \\
&= A^T P_0 + A^T \Delta P + K_0^T B^T P_0 + K_0^T B^T \Delta P + \Delta K^T B^T P_0 \\
&\quad + \Delta K^T B^T \Delta P \\
&\simeq (A+BK_0)^T P_0 + (A+BK_0)^T \Delta P + \Delta K^T B^T P_0
\end{aligned}
$$

It is assumed that P and K have small value. Thus their product can be neglected. The linearized approximation of Eq.(2.1) by neglecting second order term is considered as

$$
\begin{aligned}
& P_0 + \Delta P > 0 \\
& F_{11}(P_0, K_0) < 0 \\
& \begin{bmatrix} sin\theta G_{11}(P_0, K_0) & cos\theta G_{12}(P_0, K_0) \\ cos\theta G_{21}(P_0, K_0) & sin\theta G_{22}(P_0, K_0) \end{bmatrix} < 0
\end{aligned} \tag{2.2}
$$

where

$$
\begin{aligned}
F_{11}(P_0, K_0) =& P_0\bar{A} + \Delta P\bar{A} + \bar{A}^T P_0 + \bar{A}^T \Delta P + P_0 B \Delta K \\
& + \Delta K^T B^T P_0 + 2\gamma_0 I \\
G_{11}(P_0, K_0) =& G_{22}(P_0, K_0) \\
=& P_0\bar{A} + \Delta P\bar{A} + \bar{A}^T P_0 + \bar{A}^T \Delta P + P_0 B \Delta K \\
& + \Delta K^T B^T P_0 \\
G_{12}(P_0, K_0) =& G_{21}(P_0, K_0)^T \\
=& P_0\bar{A} + \Delta P\bar{A} - \bar{A}^T P_0 - \bar{A}^T \Delta P + P_0 B \Delta K \\
& - \Delta K^T B^T P_0
\end{aligned}
$$

and

$$\bar{A} = A + BK_0$$

Eq.(2.2) can be solved for δP and δK. Now we have to make sure that obtained solution for δP and δK satisfy Eq.(2.1).

2.3.2 Design Algorithm for Homotopy Method

1. Compute centralized controller gain K_0 and Lypunov matrix P_0 for the system by solving Eq.(2.1).

2. Let consider ΔK having same structure as K_d. $K_{0,d}$ denotes the block diagonal part of K_0. Similarly $K_{0,off}$ denotes the off block diagonal part of K_0 . Then $K_0 = K_{0,d} + K_{0,off}$. Select $\rho < \rho_{min}$, $\rho < 1$ and $\lambda = 0$.

3. Set $\bar{K}_k = K_{k-1} - \rho K_{0,off}$. Let $\bar{A} = A + BK_0$ and solve following optimization problem

$$\begin{aligned} & P_{k-1} + \Delta P < 0 \\ & F_{11}(P_{k-1}, \bar{K}_k) < 0 \\ & \begin{bmatrix} sin\theta G_{11}(P_{k-1}, \bar{K}_k) & cos\theta G_{12}(P_{k-1}, \bar{K}_k) \\ cos\theta G_{21}(P_{k-1}, \bar{K}_k) & sin\theta G_{22}(P_{k-1}, \bar{K}_k) \end{bmatrix} < 0 \end{aligned} \tag{2.3}$$

   where $\Delta P = \Delta P^T$

4. $K_k = \bar{K}_k + \Delta K$ and $P_k = P_{k-1} + \Delta P$. Check that (K_k, P_k) satisfies the condition given in Eqs.(2.1) . If not, then go to step 3 and solve for $\rho = \rho/10$ until $\rho < \rho_{min}$. If $\rho < \rho_{min}$ then we conclude that it does not give a feasible solution and requires another starting solution of centralized controller. If (K_k, P_k) satisfies Eq.(2.1) then go to step 5.

5. Set $\lambda = \lambda + \rho$. If $\lambda = 1$ go to step 6 else set $k = k + 1$ and follow step 3.

6. K_k is the required decentralized controller gain.

2.3.3 Stability of the system

Inclusion principle concept implies:

Theorem 3: If the control law designed for expanded system $\tilde{S}$ is contractible to the control law for original system S, and then the stability of closed loop expanded system (stability of subsystems and their interconnections) shows the stability of closed loop original system.

The decentralized state feedback control stabilizes the original system if the following are satisfied:

- If each subsystem satisfies pole placement constraints.
- If each subsystem follows a standard Lyapunov stability criterion which establishes the stability of the closed loop system in the expanded space.

The closed loop expanded system

$$\tilde{S}_f : \quad \dot{\tilde{x}} = (\tilde{A} + \tilde{B}\tilde{K}_D)\tilde{x} \tag{2.4}$$

The stability of closed loop expanded system can be can be verified, if all subsystems and their interconnections satisfy above stated two conditions. Individual subsystem is given by

$$S_i : \quad \tilde{A}_{fi}\tilde{x}_i, \quad \{i = 1, 2, ..n\}$$

Consider Lyapunov function

$$\tilde{V}(\tilde{x}_i) = \tilde{x}_i^T P_i \tilde{x}_i \tag{2.5}$$

There exists a symmetric positive definite matrix P_i if the subsystem is stable, which satisfies the following condition:

$$\begin{aligned} P_i &> 0 \\ \tilde{A}_{fi}^T P_i + P_i \tilde{A}_{fi} &< 0 \end{aligned} \tag{2.6}$$

and for the interconnection between subsystems

$$\begin{aligned} \tilde{V}(\tilde{x}_{ij}) &= \tilde{x}_i^T P_{ij} \tilde{x}_j \\ P_{ij} &> 0 \\ 2\tilde{A}_{fij} P_{ij} &< 0 \end{aligned} \tag{2.7}$$

Then for the whole system, consider the sum of $\tilde{V}_i(\tilde{x}_i)$

$$\tilde{V}(\tilde{x}) = \sum_{i=1}^{r} \tilde{V}_i(\tilde{x}_i) \tag{2.8}$$

Next step is to calculate time derivative of Eqs.(2.5) and (2.7) and then obtained derivatives are to calculate time derivative of $\tilde{V}(\tilde{x})$ (For the expanded system). If the expanded system is stable then the following condition should be satisfied:

$$\tilde{P}\tilde{A}_f + \tilde{A}_f^T\tilde{P} < 0 \tag{2.9}$$

where $\tilde{P}$ denotes a symmetric positive definite matrix for the closed loop expanded system.

Chapter 3

Case Study 1: Load Frequency Control of Power System

Chapter 3

Case Study 1: Load Frequency Control of Power System

Control of interconnected power systems is one the most important issue on which many research going on. The main task of power system is to provide powers according to demand of connected various loads. As load changes, frequency and tie-line power flow are shifted from its nominal value. But, deviations in both should be zero. So the system requires load frequency control.

The primary task of LFC is to keep the frequency to its nominal value against the randomly varying loads, which also known as external disturbance. The secondary task is to regulate tie-line power flows between neighboring areas. On the other hand, increase in size and complexity of the power system introduces uncertainties and disturbances in control operation. Thus it is desired that the novel control strategies be developed to achieve LFC goals and maintain reliability of the power system in an adequate level.

This chapter describes centralized and decentralized approach of controller design and its effectiveness is demonstrated with the help of an example of 2-Area Power System for both non-iterative and iterative method.

3.1 Model Description

The system model that we are going to use was derived in [20, 21].Incremental changes in demand power arises two problems: first control of real power and frequency, second control of reactive power and voltage. Both can be deal separately, here we will consider first problem.

Power equilibrium equation

The net surplus power is absorbed by the following 3 ways:

1. By increasing rate of kinetic energy W_{kin}

$$\begin{aligned}\frac{d}{dt}W_{kin} &= \frac{d}{dt}\left[W^*_{kin}\left(\frac{f}{f^*}\right)^2\right] \\ &\simeq \frac{d}{dt}\left[W^*_{kin}\left(1+2\frac{\Delta f}{f^*}\right)\right] \\ &= 2\frac{W^*_{kin}}{f^*}\frac{d}{dt}(\Delta f)\end{aligned}$$

   f^* = nominal frequency,

   W^*_{kin} = kinetic energy at frequency f^*.

2. By increasing load consumption represented by $D\Delta f$. $D = \frac{\partial P_d}{\partial f}$ MW/Hz is the rate of the system changes load at nominal frequency f^*.

3. By increasing the total export of tie-line power ΔP_{tie} .

The equilibrium equation is given by

$$\Delta P_{gi} - \Delta P_{di} = 2\frac{W^*_{kini}}{f^*}\frac{d}{dt}(\Delta f_i) + D_i\Delta f_i + \Delta P_{tiei} \tag{3.1}$$

All powers are in MW. If it is divided by rated power P_{ri}, then

$$\Delta P_{gi} - \Delta P_{di} = 2\frac{H^*_i}{f^*}\frac{d}{dt}(\Delta f_i) + D_i\Delta f_i + \Delta P_{tiei} \tag{3.2}$$

where $H_i = \frac{W^*_{kini}}{P_{ri}}$ inertia constant.

Now all powers are in per units of P_{ri}.

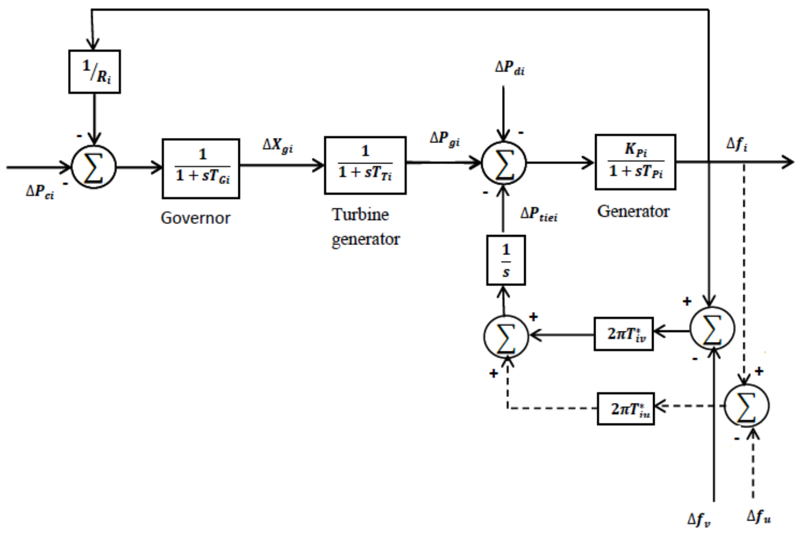


Figure 3.1: Block diagram of i^{th}-area model

Taking Laplace transformation of Eq.(3.2), then it results

$$\Delta P_{gi}(s) - \Delta P_{di}(s) = \frac{H_i}{f^*} s\Delta F_i(s) + D_i \Delta f_i(s) + \Delta P_{tiei}(s)$$

$$\left[\Delta P_{gi}(s) - \Delta P_{di}(s) - \Delta P_{tiei}(s)\right] \frac{K_{Pi}}{1 + sT_{Pi}} = \Delta F_i(s) \tag{3.3}$$

where $K_{Pi} \triangleq \frac{1}{D_i}$ Hz/pu MW

$T_{Pi} \triangleq \frac{2H_i}{f^* D_i} s$

Incremental tie-line power ΔP_{tiei}

It is defined as the total real power exported form i^{th} area is equal to the sum all outgoing powers P_{tieiv} in the lines connecting with i^{th} area and its neighbors.

$$P_{tiei} = \sum_v P_{tieiv}$$

The real power per unit transmitted through individual line

$$P_{tieiv} = \frac{|V_i| \, |V_v|}{X_{iv} P_{ri}} sin\left(\delta_i - \delta_v\right) \tag{3.4}$$

where $V_i = |V_i| \, e^{j\delta_i}$

$V_v = |V_v| \, e^{j\delta_v}$

Assuming a small phase deviations $\delta_i = \delta_i^* + \Delta\delta_i$ and $\delta_v = \delta_v^* + \Delta\delta_v$.The phase angle deviation is directly related with deviation in area frequency is given by

$$\Delta\delta_i = 2\pi \int \Delta f_i dt \tag{3.5}$$

The incremental tie-line power

$$\Delta P_{tiev} = 2\pi T_{iv}^* \left(\int \Delta f_i dt - \int \Delta f_v dt \right) \tag{3.6}$$

where $T_{iv}^* = \frac{|V_i||V_v|}{X_{iv}P_{ri}} cos\left(\delta_i^* - \delta_v^*\right)$

$$\Delta P_{tiei} = 2\pi \sum_v T_{iv}^* \left(\int \Delta f_i dt - \int \Delta f_v dt \right) \tag{3.7}$$

Taking Laplace transformation

$$\Delta P_{tiei}\left(s\right) = \frac{2\pi}{s} \sum_v T_{iv}^* \left[\Delta F_i\left(s\right) - \Delta F_v\left(s\right)\right]$$

Incremental Generated Power

A study of the generator-turbine-governor system reveals that for small changes around nominal settings the system can be represented by two time constants: T_{Gi} denotes time constant of governor and T_{Ti} denotes time lag of turbine.
The generator-turbine-governor system can be written as

$$\frac{d}{dt}\Delta P_{gi} = -\frac{1}{T_{Ti}}\Delta P_{gi} + \frac{1}{T_{Ti}}\Delta X_{gi} \tag{3.8}$$

$$\frac{d}{dt}\Delta X_{gi} = -\frac{1}{T_{Gi}}\Delta X_{gi} - \frac{1}{R_i T_{Gi}}\Delta f + \frac{1}{T_{Gi}}\Delta P_{ci} \tag{3.9}$$

Where ΔP_{gi}, ΔX_{gi} and ΔP_{ci} and are measured in pu MW, and constant R_i in Hz/pu MW. ΔP_{ci} represents incremental change in command signal for the speed changer position. ΔP_{gi} represents incremental change generated power. ΔX_{gi} represents incremental change in governor valve position.

Let us consider two-area problem tie-line deviation for the first area is proportional to tie-line deviation for the second area.

$$\Delta P_{tie2} = a_{12}\Delta P_{tie2} \tag{3.10}$$

where

$$a_{12} \triangleq -P_{r1}/P_{r2}$$

In this case there is no need to define additional state for the integral of P_{tie2}.

Two-Area Problem

For the two-area LFC problem the state vector, control input vector and perturbation are considered as

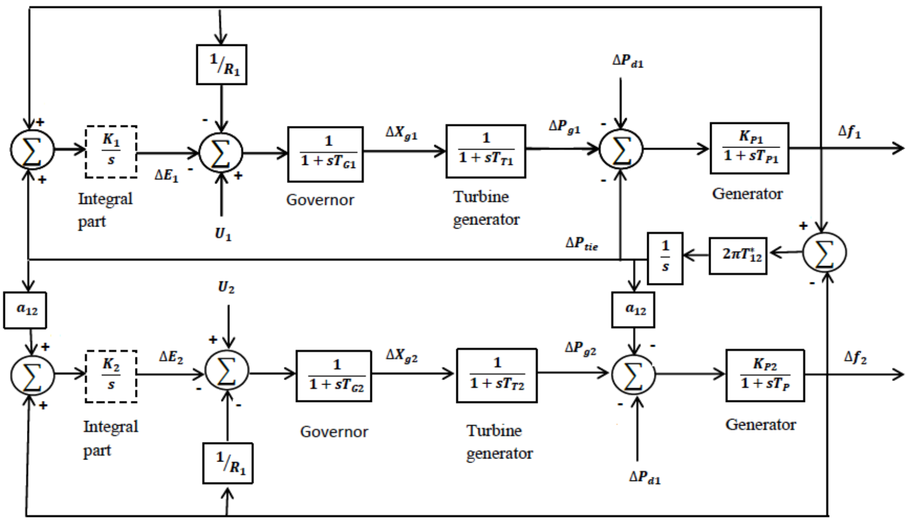


Figure 3.2: Two-Area Power System

$x^T = [\Delta f_1 \ \ \Delta P_{g1} \ \ \Delta X_{g1} \ \ \Delta E_1 \ \ \Delta P_{tie} \ \ \Delta f_2 \ \ \Delta P_{g2} \ \ \Delta X_{g2} \ \ \Delta E_2]$.

$u^T = [\Delta P_{c1} \ \ \Delta P_{c2}]$

$w^T = [\Delta P_{d1} \ \ \Delta P_{d2}]$.

where $\Delta E_i = \int \Delta P_{tiei}$

Now Two-area power system model is described by:

$$\dot{x} = Ax + Bu + \Gamma w \tag{3.11}$$

3.2 Centralized Controller Design

Consider a power system model is given by

$$\dot{x} = Ax + Bu$$

where $x \in \mathbb{R}^n$ is state vector;

$A \in \mathbb{R}^{n\times n}$ and $B \in \mathbb{R}^{n\times m}$: system matrices and

$u \in \mathbb{R}^m$ is control input.

Assumption 1: All states are available and measurable at the time of generation of control input.

The closed loop eigenvalues lie inside the desired region D if and only if there exists a positive and symmetry matrix P in such a way the optimization problem is formulated as

$$P > 0$$

$$(A+BK)P + P(A+BK)^T + 2\gamma_0 P < 0$$

$$\begin{bmatrix} sin\theta[(A+BK)P + P(A+BK)^T] & cos\theta[(A+BK)P - P(A+BK)^T] \\ cos\theta[(A+BK)P - P(A+BK)^T]^T & sin\theta[(A+BK)P + P(A+BK)^T] \end{bmatrix} < 0$$

After converting it into convex optimization problem

$$P > 0$$

$$AP + PA^T + BY + Y^T B^T + 2\gamma_0 P < 0$$

$$\begin{bmatrix} sin\theta[AP + PA^T + BY + Y^T B^T] & cos\theta[AP - PA^T + BY - Y^T B^T] \\ cos\theta[AP - PA^T + BY - Y^T B^T]^T & sin\theta[AP + PA^T + BY + Y^T B^T] \end{bmatrix} < 0$$

where $K = YP^{-1}$.

Constraints for gain optimization

$$\begin{bmatrix} -k_y I & Y^T \\ Y & -I \end{bmatrix} < 0$$

$$\begin{bmatrix} P & I \\ I & k_p I \end{bmatrix} > 0$$

$$\begin{bmatrix} \alpha I & K^T \\ K & I \end{bmatrix} > 0$$

The above optimization problem can be solved either directly or iteratively (described in chapter 2).

Example:Two-Area Power System Model

The nominal model parameters [19] are considered as

$T_{P_1} = T_{P_2} = 20sec$, $T_{T_1} = T_{T_2} = 0.3sec$, $T_{G_1} = T_{G_2} = 0.08sec$,
$K_{P_1} = K_{P_2} = 120Hz/p.u.MW$,$K_1 = K_2 = 0.401p.u.MW$ and
$R_1 = R_2 = 2.4Hz/p.u.MW$.

The System matrices are

$$A = \begin{bmatrix} -0.05 & 6 & 0 & 0 & -6 & 0 & 0 & 0 & 0 \\ 0 & -3.33 & 3.33 & 0 & 0 & 0 & 0 & 0 & 0 \\ -5.2083 & 0 & -12.5 & -12.5 & 0 & 0 & 0 & 0 & 0 \\ 0.401 & 0 & 0 & 0 & 0.401 & 0 & 0 & 0 & 0 \\ 0.545 & 0 & 0 & 0 & 0 & -0.545 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -6 & -0.05 & 6 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -3.33 & 3.33 & 0 \\ 0 & 0 & 0 & 0 & 0 & -5.2083 & 0 & -12.5 & -12.5 \\ 0 & 0 & 0 & 0 & -0.401 & 0.401 & 0 & 0 & 0 \end{bmatrix}$$

$$B^T = \begin{bmatrix} 0 & 0 & 12.5 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 12.5 & 0 \end{bmatrix}$$

$$\Gamma^T = \begin{bmatrix} -6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -6 & 0 & 0 & 0 \end{bmatrix}$$

Now employing the design algorithm presented in chapter 2 for a desired performance criterion of $\theta = 70^0$,$\gamma_0 = 2$ and $\varepsilon = 10^{-5}$ in the presence of disturbances $W^T = [0.1 \;\; 0]$.

The centralized controller gain by applying non-iterative algorithm is obtained as

$$K_{nonit} = \begin{bmatrix} -5.4283 & -5.2909 & -0.8537 & -37.3028 & 4.0066 & 3.3046 & 2.4800 & 0.6299 & 28.8349 \\ 4.0577 & 2.5639 & 0.5546 & 32.6172 & 5.6237 & -7.3616 & -6.5908 & -1.1622 & -45.8724 \end{bmatrix}$$

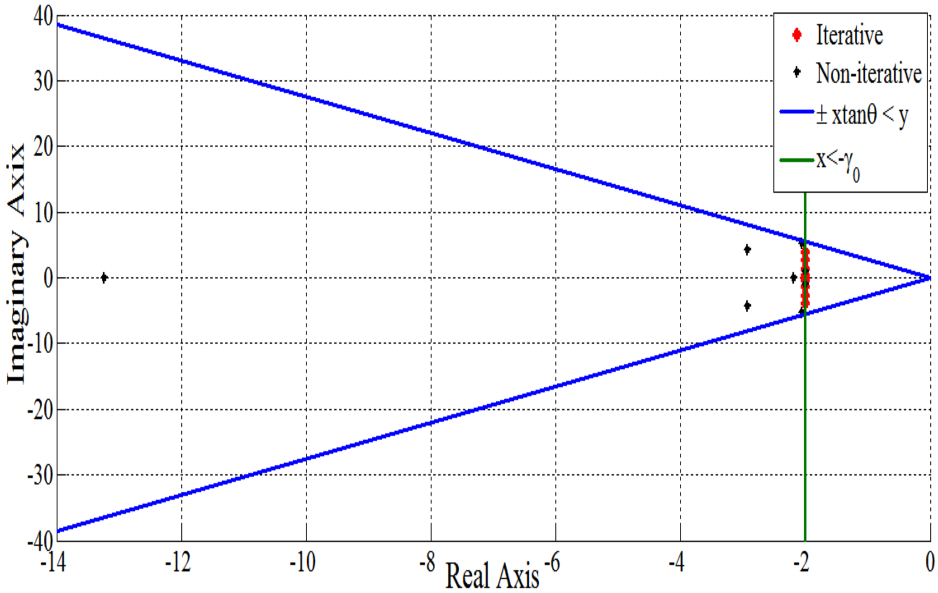


Figure 3.3: Closed loop eigen values (Centralized Control)

The above Controller gain places the closed loop eigenvalues inside the given specified region at $-13.4608 \pm 1.8856j$, $-7.2999 \pm 6.4826j$, $-3.2253 \pm 2.3887j$, $-3.1499 \pm 1.5300j$ and -2.6868 .

After appling D-K type iteration algorithm the controller gain is obtained as

$$K_{iter} = \begin{bmatrix} -0.1475 & -0.7782 & 0.5218 & -0.6691 & 0.3529 & 0.223 & 0.2297 & 0.046 & 1.04 \\ 0.1498 & 0.1469 & -0.0369 & -0.05 & 1.309 & -0.172 & -0.712 & 0.5789 & -0.1582 \end{bmatrix}$$

This places the closed loop eigenvalues at $-2.0001 \pm 3.7188j$, $-2.0001 \pm 2.7101j$, $-2.0001 \pm 1.5132j$, $-2.0008 \pm 0.3753j$ and -2.0007 .

In fig.(3.3) the location of closed loop eigenvalues for both the case iterative and non-iterative are presented. We can observe that the closed loop poles in iterative algorithm are nearer than non-iterative.

The control input u_1 and u_2 are presented in fig.(3.4) for both algorithm. The simulation results for the responses of the system are shown in fig.(3.5).

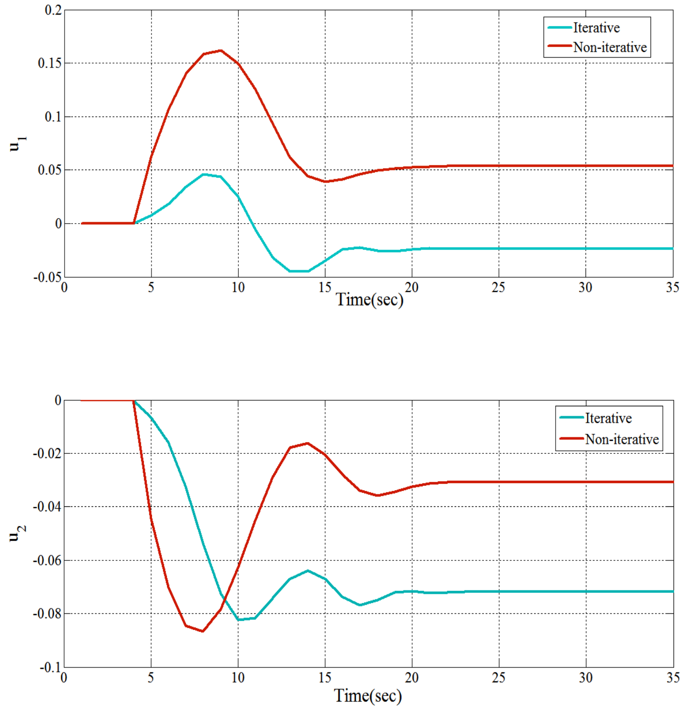


Figure 3.4: Control inputs u_1 and u_2(Centralized Control)

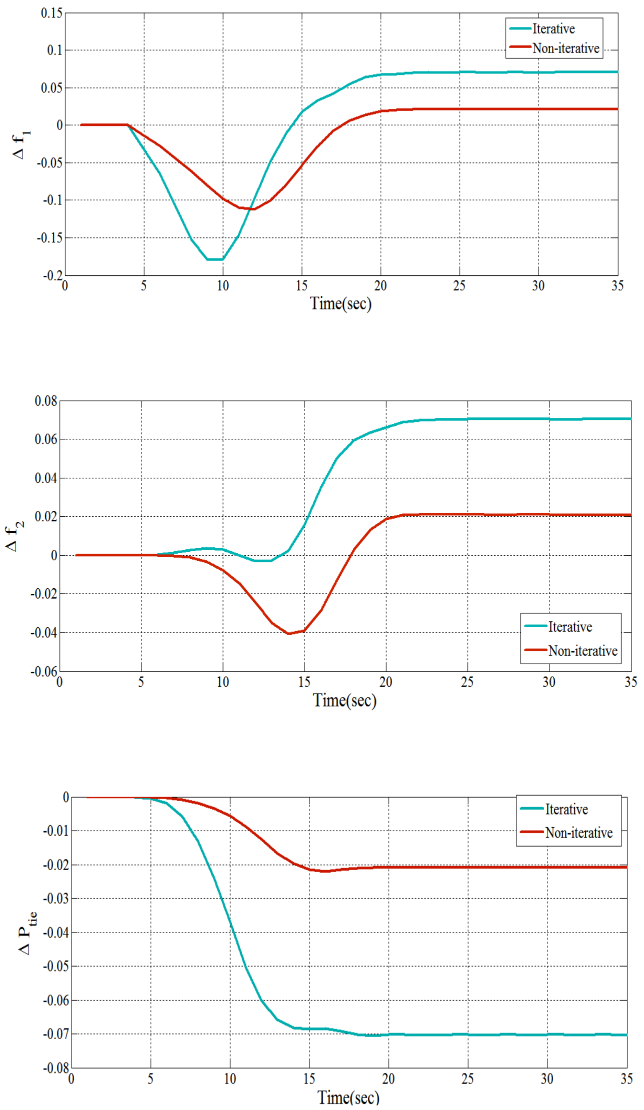


Figure 3.5: Responses of the system in the presence step disturbance input(Centralized Control)

3.3 Decentralized Controller Design

An N-area power system LFC can be modeled as a large-scale power system consisting of N subsystem:

$$\dot{x} = A_N x + B_N u$$

where $u = [u_1, ...u_N]^T$ is control input; $x = [x_1, ...x_N]^T$ and x_i are the state variables for the i^{th} area. In another way

$$\dot{x_i} = A_{ii} x_i + \sum_{j=1, j\neq i}^{N} A_{ij} x_j + B_{ii} u_i$$

It is assume that all (A_{ii}, B_{ii}) are controllable. A_{ij} term included due to interconnection of one area to other.

Here control problem is to design N decentralized local controller or equivalently the design of an $N \times N$ block diagonal matrix $K_D = diag[k_1, ...k_N]$.

If we combine above control problem with pole placement, then the problems becomes to design of an $N \times N$ block diagonal matrix $K_D = diag[k_1, ...k_N]$ such that the closed loop poles lie inside the desired region with minimum gain and restricts the poles near the boundary.

The optimization problem

$$P_D > 0$$

$$(A + B_D K_D)P_D + P_D(A + B_D K_D)^T + 2\gamma_0 P_D < 0$$

$$\begin{bmatrix} sin\theta[(A + B_D K_D)P_D + P_D(A + B_D K_D)^T] & cos\theta[(A + B_D K_D)P_D - P_D(A + B_D K_D)^T] \\ cos\theta[(A + B_D K_D)P_D - P_D(A + B_D K_D)^T]^T & sin\theta[(A + B_D K_D)P_D + P_D(A + B_D K_D)^T] \end{bmatrix} < 0$$

The LMI optimization problem

$$P_D > 0$$

$$AP_D + P_D A^T + B_D Y_D + Y^T B^T + 2\gamma_0 P < 0$$

$$\begin{bmatrix} sin\theta[AP_D + P_D A^T + B_D Y_D + Y_D^T B_D^T] & cos\theta[AP_D - P_D A^T + B_D Y_D - Y_D^T B_D^T] \\ cos\theta[AP_D - P_D A^T + B_D Y_D - Y_D^T B_D^T]^T & sin\theta[AP_D + P_D A^T + B_D Y_D + Y_D^T B_D^T] \end{bmatrix} < 0$$

where we consider $P_D = P_D^{-1}$ and $K_D = Y_D P_D^{-1}$.

Constraints for gain optimization

$$\begin{bmatrix} -k_y I & Y_D^T \\ Y_D & -I \end{bmatrix} < 0$$

$$\begin{bmatrix} P_D & I \\ I & k_p I \end{bmatrix} > 0$$

$$\begin{bmatrix} \alpha I & K_D^T \\ K_D & I \end{bmatrix} > 0$$

The above optimization problem can be solved by employing non-iterative and iterative algorithm as discribed in chapter 2.

Example

The Power System model with 2 decentralized subsystem is described by:

$$\dot{x} = Ax + B_D u_D$$

where$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$ and $B_D = diag[B_{11}, B_{22}]$.

Assumption 2: The system exists no decentralized fixed mode.

If the system exists no decentralized fixed mode then we can directly decompose the system into subsystem. The block diagonal controller gain

$$K_D = [k_{11}, k_{22}]$$

The same specifications for power system as used in centralized structure, are considered in decentralized controller structure except information structure constraints.

The decentralized controller gain

$$K_{Dnonit} = \begin{bmatrix} -102.38 & -35.28 & -3.26 & -8832.97 & 4998 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -4441 & -680.8 & -26.66 & -45371 \end{bmatrix}$$

The closed loop eigenvalues are placed at $-48.4 \pm 40.35j, -11.27 \pm 18.64j, -17.26 \pm 10.09j, -2.03 \pm 4.40j$ and -247.82

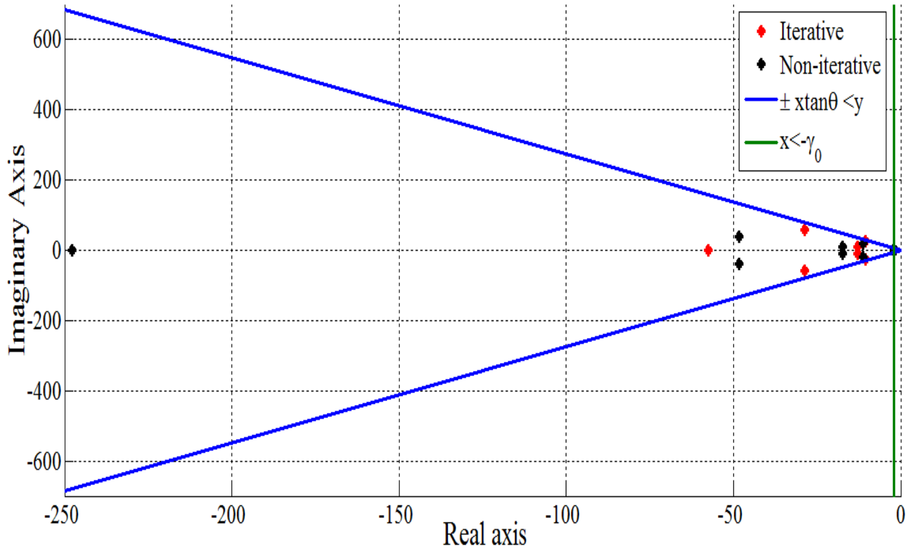


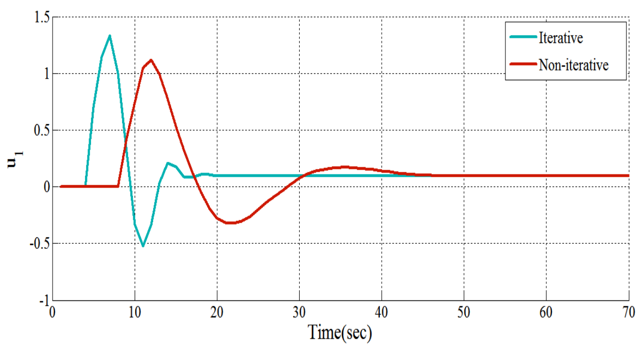
Figure 3.6: Closed loop eigen values (Decentralized Control)

In the case of iterative algorithm the decentralized controller gain

$$K_{Diter} = \begin{bmatrix} -108.5926 & -35.95 & -2.44 & -10187.7 & 5824 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1110.7 & -184 & -8.29 & -11685.8 \end{bmatrix}$$

The closed loop eigenvalues are placed at $-28.69 \pm 58.24j, -10.53 \pm 27.17j, -12.9 \pm 9.24j, -2.03 \pm 4.57j$ and -57.5. By observing the location of closed loop eigen values, we can conclude that the designed decentralized state feedback controller stabilizes the system.

For decentralized approach of controller design the closed loop eigen values are shown in fig.(3.6) for both non-iterative and iterative algorithm. The control inputs u_1 and u_2 are given in fig.(3.7) which are obtained by using controller gain K_{Dnonit} and K_{Diter}. The responses of the system are shown in fig.(3.8).



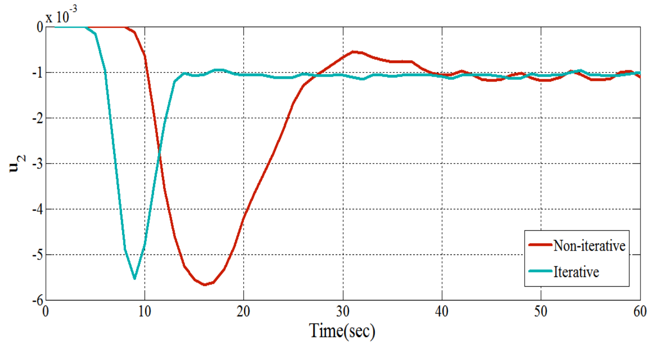


Figure 3.7: Control inputs u_1 and u_2(Decentralized Control)

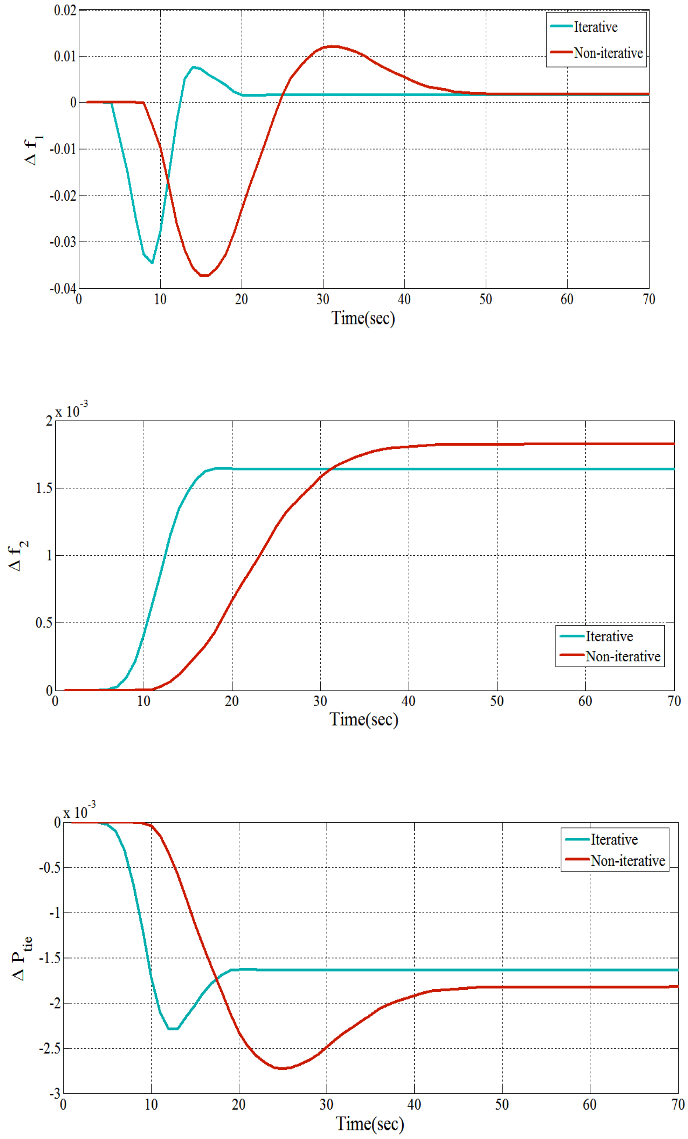


Figure 3.8: Responses of the system in the presence step disturbance input(Decentralized Control)

Chapter 4

Case Study 2: Formation control of Unmanned Aerial Vehicles

Chapter 4

Case Study 2: Formation control of unmanned aerial vehicles

Formation control is one of the challenging problems in control engineering field for controlling a group of unmanned aerial vehicles (UAVs). In many applications a group of UAVs follows a predefined trajectory while maintaining desired formation. There are three basic approaches for formation control: behavior-based, virtual structure and leader following. Here we are going to attempt leader following in which one or more vehicles may be leader while other followers.

Formation control has wide range of applications. For example, in military operations a group of AUVs are used for target vertical damage assessment and reconnaissance, in civilian works such as vegetation growth analysis. In automated highway system, the efficiency of transportation network can be increased if the vehicles form a desired pattern at desired velocity while maintaining a specified distances between vehicles.

UAVs fly in formation is better than conventional systems, such as it can reduce system cost, increase the robustness and efficiency, and provide rapid configurability and structural flexibility (for decentralized control schemes).

This chapter describes decentralized approach of controller design. Each vehicle is modeled as kinematic model, and an information structure constraint is assumed in which each vehicle except the leading one has the state information of vehicle in front of it. This information structure constraint is considered minimal because there is one communication link between vehicles. The resulted system is treated as interconnected system with overlapped subsystems.

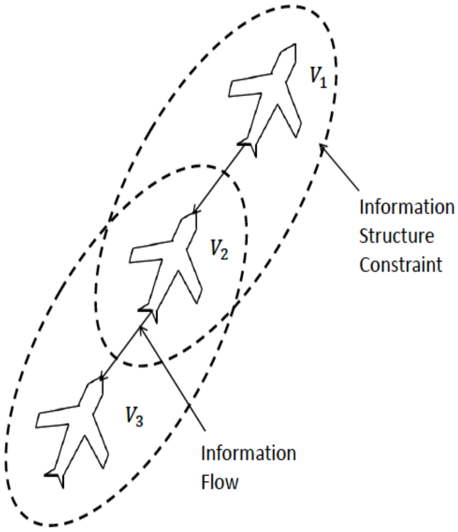


Figure 4.1: A platoon with leader-follower type formation

4.1 Model Description

The kinematic model for a single vehicle is given by:

$$\dot{X} = v cos\psi$$

$$\dot{Y} = v sin\psi$$

$$\dot{\psi} = \omega \tag{4.1}$$

Where X and Y are co-ordinates

ψ is the heading angle in the $XY-$plane

The speed v and angular turn rate ω are considered as control inputs.

The kinematic model of single vehicle is nonlinear so first it needs to linearize, which results a singular matrix. In order to solve this problem consider speed v as new state variable and acceleration a as a new input variable.

The new state and input variables are defined as

$$\xi = \begin{bmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \\ \xi_4 \end{bmatrix} = \begin{bmatrix} X \\ Y \\ \psi \\ v \end{bmatrix}, \quad \eta = \begin{bmatrix} \eta_1 \\ \eta_2 \end{bmatrix} = \begin{bmatrix} a \\ \omega \end{bmatrix} \tag{4.2}$$

Now the kinematic model (4.1) can be given as

$$\begin{bmatrix} \dot{\xi}_1 \\ \dot{\xi}_2 \\ \dot{\xi}_3 \\ \dot{\xi}_4 \end{bmatrix} = \begin{bmatrix} \xi_4 cos\left(\xi_3\right) \\ \xi_4 sin\left(\xi_3\right) \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} a \\ \omega \end{bmatrix} \tag{4.3}$$

Or $$\dot{\xi} = f\left(\xi\right) + g\left(\xi\right)\eta$$

By introducing change of state variables as $z = T(\xi)$such that

$$\begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix} = \begin{bmatrix} \xi_1 \\ \xi_2 \\ \xi_4 cos\left(\xi_3\right) \\ \xi_4 sin\left(\xi_3\right) \end{bmatrix} \tag{4.4}$$

and change of input variables as $\eta = M\left(\xi\right)u$ where $u \in \mathbb{R}^2$ is a new input variable, where

$$M\left(\xi\right) = \begin{bmatrix} cos\left(\xi_3\right) & sin\left(\xi_3\right) \\ -sin\left(\xi_3\right)/\xi_4 & cos\left(\xi_3\right)/\xi_4 \end{bmatrix} \tag{4.5}$$

The transformation in (4.4) and (4.5) follow the following linearization of (4.3)

$$\dot{z} = \frac{\partial T}{\partial \xi}\dot{\xi} \Rightarrow \dot{z} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} z + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} u \tag{4.6}$$

Or

$$\dot{z} = Ez + Fu$$

We can write as

$$\dot{z} = \begin{bmatrix} 0_2 & I_2 \\ 0_2 & 0_2 \end{bmatrix} z + \begin{bmatrix} 0_2 \\ I_2 \end{bmatrix} u \tag{4.7}$$

where $z \in \mathbb{R}^4$ and $u \in \mathbb{R}^2$ represent state and input of the system respectively.For simplification 0_2 (2×2 zero matrix) and I_2 (2×2 identity matrix) will be denoted as 0 and I respectively.

4.2 Problem Formulation

By observing (4.4), we find that it contains the following 2 types of variables, which are used to control the vehicle in the formation.

- Variables that represent position co-ordinates of a vehicle.
- Variables that represent speed co-ordinates of a vehicle.

Let us decompose the state variables of i^{th} vehicle according to above:

$$z_i = \begin{bmatrix} z_i^d \\ z_i^s \end{bmatrix} \in \mathbb{R}^4 \tag{4.8}$$

where

$$z_i^d = \begin{bmatrix} z_{i1} \\ z_{i2} \end{bmatrix} = \begin{bmatrix} X_i \\ Y_i \end{bmatrix}$$
$$z_i^s = \begin{bmatrix} z_{i3} \\ z_{i4} \end{bmatrix} = \begin{bmatrix} v_i cos\psi_i \\ v_i sin\psi_i \end{bmatrix}$$

By imposing information structure constraint, in which each vehicle except the leading one has state information about the vehicle in front of it. The formation shown in fig.(4.1) has 1 platoon having 3 vehicles .

Note:

- The leading vehicle does not receive any information from the vehicles behind it. Thus its dynamics are governed independently.
- To control distances between vehicles, position of leading vehicle does not require.

Consider a platoon of r vehicles, now by introducing a change of variables such that

$$e_1^s = z_1^s - v_{d1} \qquad \text{For leading vehicle}$$
$$\begin{Bmatrix} e_i^d = z_{i-1}^d - z_i^d - d_{i-1} \\ e_i^s = z_i^s - v_{di} \end{Bmatrix}, \qquad i \in \{2, ..., r\} \tag{4.9}$$

where $d_{i-1} \in \mathbb{R}^2$ is a constant desired Euclidean distance between the $(i-1)^{st}$ and i^{th} vehicles and $v_{di} \in \mathbb{R}^2$ is the desired speed for the i^{th} vehicles. For maintaining the desired distances between vehicles the desired speed should be same for each vehicle.

$$v_{di} = v_d, \qquad i \in \{2, ..., r\} \tag{4.10}$$

then

$$\dot{e}_1^s = u_1, \qquad \text{For leading vehicle}$$
$$\begin{Bmatrix} \dot{e}_i^d = e_{i-1}^s - e_i^s \\ \dot{e}_i^s = u_i \end{Bmatrix}, \qquad i \in \{2, ..., r\} \tag{4.11}$$

Where $u_i \in \mathbb{R}^2$ is control input for i^{th} vehicle. The above can be considered as interconnected system in which each subsystem has the following state variables:

$$e_1 = e_1^s, \quad e_i = \begin{bmatrix} e_i^d \\ e_i^s \end{bmatrix} \quad \text{For all } i \in \{2, ..., r\} \tag{4.12}$$

In fig $r = 3$, then Eq.(4.11) can be written as

$$\begin{bmatrix} \dot{e}_1^s \\ \dot{e}_2^d \\ \dot{e}_2^s \\ \dot{e}_3^d \\ \dot{e}_3^s \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ I & 0 & -I & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & I & 0 & -I \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} e_1^s \\ e_2^d \\ e_2^s \\ e_3^d \\ e_3^s \end{bmatrix} + \begin{bmatrix} I & 0 & 0 \\ 0 & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & 0 \\ 0 & 0 & I \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} \tag{4.13}$$

or
$$\dot{e} = Ae + Bu$$

The generalized form can be written as

$$\begin{bmatrix} \dot{x}_2^1 \\ \dot{x}_1^2 \\ \dot{x}_2^2 \\ \dot{x}_1^3 \\ \dot{x}_2^3 \end{bmatrix} = \begin{bmatrix} A_{22}^1 & 0 & 0 & 0 & 0 \\ A_{12}^{21} & A_{11}^2 & A_{12}^2 & 0 & 0 \\ 0 & A_{21}^2 & A_{22}^2 & 0 & 0 \\ 0 & 0 & A_{12}^{32} & A_{11}^3 & A_{12}^3 \\ 0 & 0 & 0 & A_{21}^3 & A_{22}^3 \end{bmatrix} \begin{bmatrix} x_2^1 \\ x_1^2 \\ x_2^2 \\ x_1^3 \\ x_2^3 \end{bmatrix} + \begin{bmatrix} B_{22}^1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & B_{22}^2 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & B_{22}^3 \end{bmatrix} \begin{bmatrix} u^1 \\ u^2 \\ u^3 \end{bmatrix} \tag{4.14}$$

where the state vector $x \in \mathbb{R}^{10}$ and partition into $x^T = [x_2^1 \; x_1^2 \; x_2^2 \; x_1^3 \; x_2^3]$ with dimensions $n_i = 2$, $i = 1, 2, 3, 4, 5$ respectively. Input vector $u \in \mathbb{R}^6$ and $u^T = [u^1 \; u^2 \; u^3]$ with dimensions $m_i = 2$, $i = 1, 2, 3$ respectively.

4.3 Decentralized overlapping control

According to information structure constraint, subsystem in Eq.(4.13) can be decoupled by the following expansion:

$$
\begin{aligned}
&\tilde{e}_1 = e_1^s, \quad \tilde{u}_1 = u_1, \\
&\tilde{e}_i = \begin{bmatrix} \tilde{e}_{i-1}^s \\ \tilde{e}_i^d \\ \tilde{e}_i^s \end{bmatrix}, \qquad \tilde{u}_i = \begin{bmatrix} u_{i-1} \\ u_i \end{bmatrix}, \qquad i \in \{2, ..., r\}
\end{aligned}
\tag{4.15}
$$

for $r = 3$ the original system (4.14) can be decomposed into the following subsystems

$$
\begin{bmatrix} \dot{x}_2^1 \\ \dot{x}_1^2 \\ \dot{x}_2^2 \\ \dot{x}_1^3 \\ \dot{x}_2^3 \end{bmatrix} = \begin{bmatrix} A_{22}^1 & 0 & 0 & 0 & 0 \\ A_{12}^{21} & A_{11}^2 & A_{12}^2 & 0 & 0 \\ 0 & A_{21}^2 & A_{22}^2 & 0 & 0 \\ 0 & 0 & A_{12}^{32} & A_{11}^3 & A_{12}^3 \\ 0 & 0 & 0 & A_{21}^3 & A_{22}^3 \end{bmatrix} \begin{bmatrix} x_2^1 \\ x_1^2 \\ x_2^2 \\ x_1^3 \\ x_2^3 \end{bmatrix} + \begin{bmatrix} B_{22}^1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & B_{22}^2 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & B_{22}^3 \end{bmatrix} \begin{bmatrix} u_2^1 \\ u_2^2 \\ u_2^3 \end{bmatrix}
$$

$$
\begin{aligned}
S_1 : &\qquad \dot{x}_2^1 = A_{22}^1 x_2^1 + B_{22}^1 u_2^1 \\
S_2 : &\qquad \begin{bmatrix} \dot{x}_2^1 \\ \dot{x}_1^2 \\ \dot{x}_2^2 \end{bmatrix} = \begin{bmatrix} A_{22}^1 & 0 & 0 \\ A_{12}^{21} & A_{11}^2 & A_{12}^2 \\ 0 & A_{21}^2 & A_{22}^2 \end{bmatrix} \begin{bmatrix} x_2^1 \\ x_1^2 \\ x_2^2 \end{bmatrix} + \begin{bmatrix} B_{22}^1 & 0 \\ 0 & 0 \\ 0 & B_{22}^2 \end{bmatrix} \begin{bmatrix} u^1 \\ u^2 \end{bmatrix} \\
S_3 : &\qquad \begin{bmatrix} \dot{x}_2^2 \\ \dot{x}_1^3 \\ \dot{x}_2^3 \end{bmatrix} = \begin{bmatrix} A_{22}^2 & 0 & 0 \\ A_{12}^{32} & A_{11}^3 & A_{12}^3 \\ 0 & A_{21}^3 & A_{22}^3 \end{bmatrix} \begin{bmatrix} x_2^2 \\ x_1^3 \\ x_2^3 \end{bmatrix} + \begin{bmatrix} B_{22}^2 & 0 \\ 0 & 0 \\ 0 & B_{22}^3 \end{bmatrix} \begin{bmatrix} u^2 \\ u^1 \end{bmatrix}
\end{aligned}
\tag{4.16}
$$

The expanded system $\tilde{S}$ can be written as

$$
\begin{bmatrix} \dot{\tilde{x}}_1 \\ \dot{\tilde{x}}_2 \\ \dot{\tilde{x}}_3 \end{bmatrix} = \begin{bmatrix} A_{22}^1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & A_{22}^1 & 0 & 0 & 0 & 0 & 0 \\ 0 & A_{12}^{21} & A_{11}^2 & A_{12}^2 & 0 & 0 & 0 \\ 0 & 0 & A_{21}^2 & A_{22}^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & A_{22}^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & A_{12}^{32} & A_{11}^3 & A_{12}^3 \\ 0 & 0 & 0 & 0 & 0 & A_{21}^3 & A_{22}^3 \end{bmatrix} \begin{bmatrix} \tilde{x}_1 \\ \tilde{x}_2 \\ \tilde{x}_3 \end{bmatrix} + \begin{bmatrix} B_{22}^1 & 0 & 0 & 0 & 0 \\ 0 & B_{22}^1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & B_{22}^2 & 0 & 0 \\ 0 & 0 & 0 & B_{22}^2 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & B_{22}^3 \end{bmatrix} \begin{bmatrix} \tilde{u}_1 \\ \tilde{u}_2 \\ \tilde{u}_3 \end{bmatrix}
\tag{4.17}
$$

Or

$$\dot{\tilde{x}} = \tilde{A}_D \tilde{x} + \tilde{B}_D \tilde{u}$$

where $\tilde{x}_1 = x_2^1$, $\tilde{x}_2^T = [x_2^1 \ x_1^2 \ x_2^2]$, $\tilde{x}_3^T = [x_2^2 \ x_1^3 \ x_2^3]$, $\tilde{u}_1 = u^1$, $\tilde{u}_2^T = [u^1 \ u^2]$ and $\tilde{u}_3^T = [u^2 \ u^3]$

The expansion and contraction matrices are given as

$$V = \begin{bmatrix} I & 0 & 0 & 0 & 0 \\ I & 0 & 0 & 0 & 0 \\ 0 & I & 0 & 0 & 0 \\ 0 & 0 & I & 0 & 0 \\ 0 & 0 & I & 0 & 0 \\ 0 & 0 & 0 & I & 0 \\ 0 & 0 & 0 & 0 & I \end{bmatrix}, U = \begin{bmatrix} \frac{1}{2}I & \frac{1}{2}I & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & I & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2}I & \frac{1}{2}I & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & I & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & I \end{bmatrix},$$

$$R = \begin{bmatrix} I & 0 & 0 \\ I & 0 & 0 \\ 0 & I & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix}, Q = \begin{bmatrix} \frac{1}{2}I & \frac{1}{2}I & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2}I & \frac{1}{2}I & 0 \\ 0 & 0 & 0 & 0 & I \end{bmatrix} \tag{4.18}$$

Using Eqs.(4.14) and (4.16)-(4.17) we can easily verify the inclusion principle. Employing the state feedback control laws, then from Eqs. (4.9) and (4.16) the information structure constraint in expanded space can be described as

$$\tilde{K}_D = \begin{bmatrix} (\tilde{K}_{22}^1)_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & (\tilde{K}_{22}^1)_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & \tilde{K}_{22}^{21} & \tilde{K}_{21}^2 & (\tilde{K}_{22}^2)_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & (\tilde{K}_{22}^2)_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & \tilde{K}_{22}^{32} & \tilde{K}_{21}^3 & \tilde{K}_{22}^3 \end{bmatrix} \tag{4.19}$$

where $\tilde{K}^i \in \mathbb{R}^{2\times 2}$ for $i \in \{1, 2\}$. Thus dimension of $\tilde{K}_D$ is 10×14.The above controller is decentralized, but can not be implemented in the original space after contraction. So it needs some modification [4].

$$\tilde{K}_{DM} = \begin{bmatrix} \frac{(\tilde{K}_{22}^1)_1+(\tilde{K}_{22}^1)_2}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{(\tilde{K}_{22}^1)_1+(\tilde{K}_{22}^1)_2}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & \tilde{K}_{22}^{21} & \tilde{K}_{21}^2 & \frac{(\tilde{K}_{22}^2)_1+(\tilde{K}_{22}^2)_2}{2} & 0 & 0 & 0 \\ 0 & \tilde{K}_{22}^{21} & \tilde{K}_{21}^2 & 0 & \frac{(\tilde{K}_{22}^2)_1+(\tilde{K}_{22}^2)_2}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \tilde{K}_{22}^{32} & \tilde{K}_{21}^3 & \tilde{K}_{22}^3 \end{bmatrix} \tag{4.20}$$

The above modification will preserved the stability of closed loop system since both matrix $\tilde{K}_D$ and $\tilde{K}_{DM}$ have same main diagonal blocks.

Subsystems S_2 and S_3 can be assumed to have same dynamics and each leader governs its dynamics independently. Thus we can put the following constraints

$$(\tilde{K}_{22}^1)_1 = (\tilde{K}_{22}^1)_2 = (\tilde{K}_{22}^2)_1 = (\tilde{K}_{22}^2)_2 = \tilde{K}_{22}^3 = \tilde{K}_1,$$
$$\tilde{K}_{22}^{21} = \tilde{K}_{22}^{32} = \tilde{K}_2 \text{ and}$$
$$\tilde{K}_{21}^2 = \tilde{K}_{21}^3 = \tilde{K}_3.$$

The controller in the expanded space after modification

$$\tilde{K}_{DM} = \begin{bmatrix} \tilde{K}_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \tilde{K}_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \tilde{K}_2 & \tilde{K}_3 & \tilde{K}_1 & 0 & 0 & 0 \\ 0 & \tilde{K}_2 & \tilde{K}_3 & 0 & \tilde{K}_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & \tilde{K}_2 & \tilde{K}_3 & \tilde{K}_1 \end{bmatrix} \tag{4.21}$$

Then, the feedback stabilizing gain which satisfy the inclusion principle in original space is given as

$$K_D = \begin{bmatrix} \tilde{K}_1 & 0 & 0 & 0 & 0 \\ \tilde{K}_2 & \tilde{K}_3 & \tilde{K}_1 & 0 & 0 \\ 0 & 0 & \tilde{K}_2 & \tilde{K}_3 & \tilde{K}_1 \end{bmatrix} \tag{4.22}$$

If we solve for individual subsystem then contol law

$$\tilde{u}_i = \begin{bmatrix} \tilde{K}_1 & 0 & 0 \\ \tilde{K}_2 & \tilde{K}_3 & \tilde{K}_1 \end{bmatrix} \tilde{x}_i \tag{4.23}$$

Or

$$\tilde{u}_i = \tilde{K}_d \tilde{x}_i$$

Let us combine decentralized approach with pole placement method then the optimization problem is given by

Subject to $\tilde{P} > 0$

$$(\tilde{A} + \tilde{B}\tilde{K}_d)\tilde{P} + \tilde{P}(\tilde{A} + \tilde{B}\tilde{K}_d)^T + 2\gamma_0\tilde{P} < 0$$

$$\begin{bmatrix} sin\theta[(\tilde{A} + \tilde{B}\tilde{K}_d)\tilde{P} + \tilde{P}(\tilde{A} + \tilde{B}\tilde{K}_d)^T] & cos\theta[(\tilde{A} + \tilde{B}\tilde{K}_d)\tilde{P} - \tilde{P}(\tilde{A} + \tilde{B}\tilde{K}_d)^T] \\ cos\theta[(\tilde{A} + \tilde{B}\tilde{K}_d)\tilde{P} - \tilde{P}(\tilde{A} + \tilde{B}\tilde{K}_d)^T]^T & sin\theta[(\tilde{A} + \tilde{B}\tilde{K}_d)\tilde{P} + \tilde{P}(\tilde{A} + \tilde{B}\tilde{K}_d)^T] \end{bmatrix} < 0 \tag{4.24}$$

and its linearized form

$$
\begin{aligned}
&\text{Subject to } \tilde{P}_d > 0 \\
&\tilde{A}\tilde{P}_d + \tilde{P}_d\tilde{A}^T + \tilde{B}\tilde{Y}_d + \tilde{Y}_d^T\tilde{B}^T + 2\gamma_0\tilde{P}_d < 0 \\
&\begin{bmatrix} sin\theta[\tilde{A}\tilde{P}_d + \tilde{P}_d\tilde{A}^T + \tilde{B}\tilde{Y}_d + \tilde{Y}_d^T\tilde{B}^T] & cos\theta[\tilde{A}\tilde{P}_d - \tilde{P}_d\tilde{A}^T + \tilde{B}\tilde{Y}_d - \tilde{Y}_d^T\tilde{B}^T] \\ cos\theta[\tilde{A}\tilde{P}_d - \tilde{P}_d\tilde{A}^T + \tilde{B}\tilde{Y}_d - \tilde{Y}_d^T\tilde{B}^T]^T & sin\theta[\tilde{A}\tilde{P}_d + \tilde{P}_d\tilde{A}^T + \tilde{B}\tilde{Y}_d + \tilde{Y}_d^T\tilde{B}^T] \end{bmatrix} < 0
\end{aligned}
\tag{4.25}
$$

The matrix variables $\tilde{Y}_d$ and $\tilde{P}_d$ with imposed structure

$$
\tilde{Y}_d = \begin{bmatrix} \tilde{Y}_1 & 0 & 0 \\ \tilde{Y}_2 & \tilde{Y}_3 & \tilde{Y}_1 \end{bmatrix}, \quad \tilde{P}_d = \begin{bmatrix} \tilde{P}_1 & 0 & 0 \\ 0 & \tilde{P}_2 & 0 \\ 0 & 0 & \tilde{P}_1 \end{bmatrix} \tag{4.26}
$$

The structure of $\tilde{Y}_d$ and $\tilde{P}_d$ in Eq.(4.25) guarantees that $\tilde{K} = \tilde{Y}_d\tilde{P}_d^{-1}$ will give same structure as in Eq.(4.23).

It is important to note for the structure of $\tilde{Y}_d$ and $\tilde{P}_d$ Eq.(4.25) will not result a feasible solution. This problem can be solved by using Homotopy method for decentralized control design as described in chapter 2.

4.4 Example

Let us consider a platoon of 3 vehicles flying in the formation as shown in fig.(4.1). Consider nominal speed v_d is $[300, 0]$[ft/s] and the desired distances between vehicles $d = [400, 400]^T$[ft]. The design algorithm as described in chapter 2 is applied to obtain decentralized static feedback controller. Assuming $\gamma_0 = 1$ and $\theta = 60°$.

4.4.1 Non-iterative Algorithm

The stabilizing controller gain in the expanded system $\tilde{S}$ are obtained as

$$\tilde{K}_D = \begin{bmatrix} \tilde{K}_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \tilde{K}_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \tilde{K}_2 & \tilde{K}_3 & \tilde{K}_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \tilde{K}_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & \tilde{K}_2 & \tilde{K}_3 & \tilde{K}_1 \end{bmatrix}$$

Where

$$\tilde{K}_1 = \begin{bmatrix} -121767754 & 0 \\ 0 & -118602737 \end{bmatrix}$$

$$\tilde{K}_2 = \begin{bmatrix} -32006626 & 0 \\ 0 & -31174702 \end{bmatrix}$$

$$\tilde{K}_3 = \begin{bmatrix} 312433809 & 0 \\ 0 & 304312954 \end{bmatrix}$$

After modifying $\tilde{K}_D$ as in Eq.(4.21)

$$\tilde{K}_{DM} = \begin{bmatrix} \tilde{K}_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \tilde{K}_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \tilde{K}_2 & \tilde{K}_3 & \tilde{K}_1 & 0 & 0 & 0 \\ 0 & \tilde{K}_2 & \tilde{K}_3 & 0 & \tilde{K}_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & \tilde{K}_2 & \tilde{K}_3 & \tilde{K}_1 \end{bmatrix}$$

Then, the controller gain after contraction which stabilizes the system in the original space

$$K_{Dnonit} = \begin{bmatrix} \tilde{K}_1 & 0 & 0 & 0 & 0 \\ \tilde{K}_2 & \tilde{K}_3 & \tilde{K}_1 & 0 & 0 \\ 0 & 0 & \tilde{K}_2 & \tilde{K}_3 & \tilde{K}_1 \end{bmatrix}$$

In the original space the controller places closed loop eigen values at the following locations

4 poles at -2.57,

2 poles at -121767751,

2 poles at -121767754,

1 poles at -118602737,

1 poles at $-118602734 + 0.0001j$ and

1 poles at $-118602734 - 0.0001j$

The simulation result is shown in fig.(4.2) for non-iterative algorithm by considering one set of initial conditions. Position co-ordinates are in feet.

Horizontal distances between vehicles V_1 and V_2, and V_2 and V_3 for non-iterative algorithm by considering one set of initial conditions are shown in fig.(4.3). The distances are in feet. In fig(4.4), the errors in speed of vehicles V_1, V_2 and V_3 are shown. Speeds are in [ft/s].

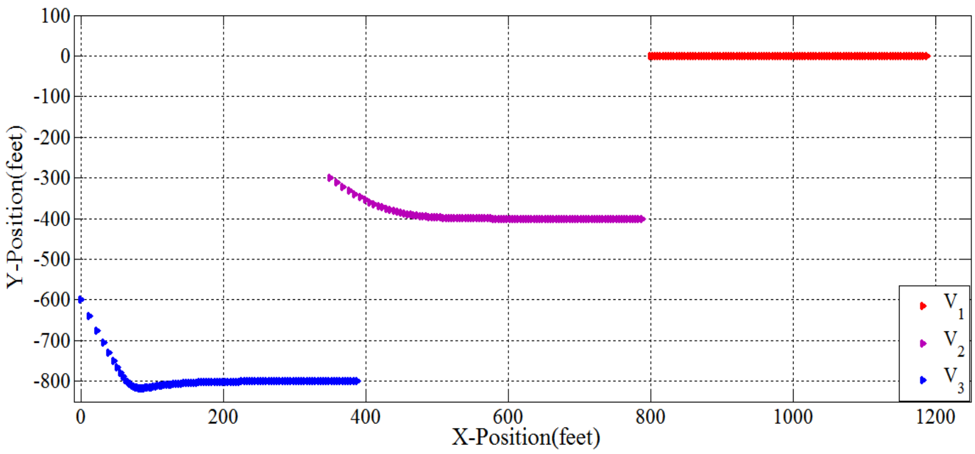


Figure 4.2: Snapshots of the formation for one set of initial conditions - Decentralized (Non-iterative algorithm)

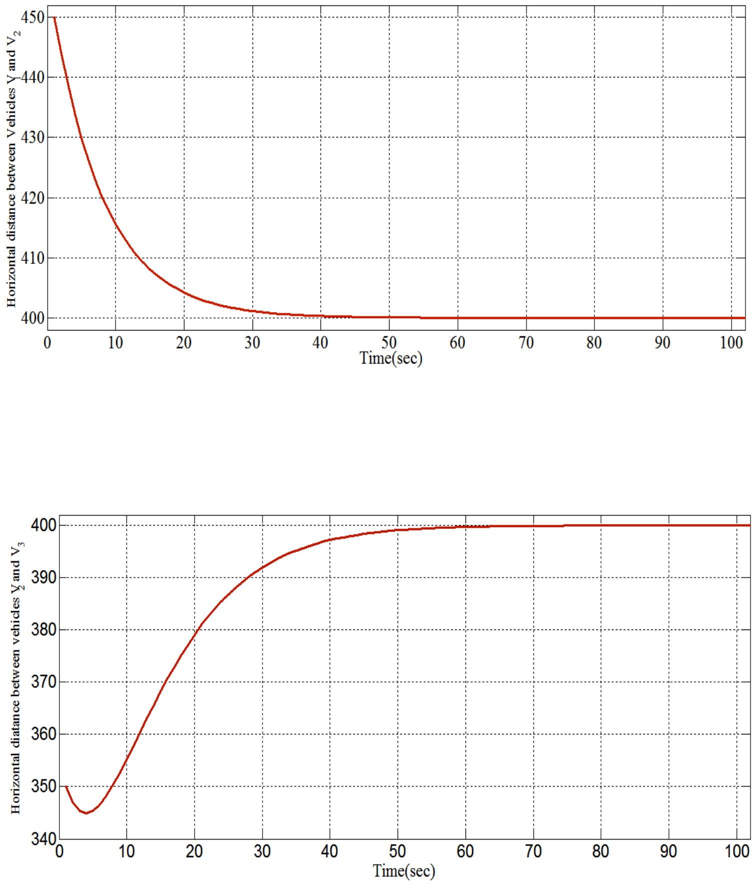


Figure 4.3: Horizontal distances between vehicles V_1 and V_2, and V_2 and V_3 (Non-iterative)

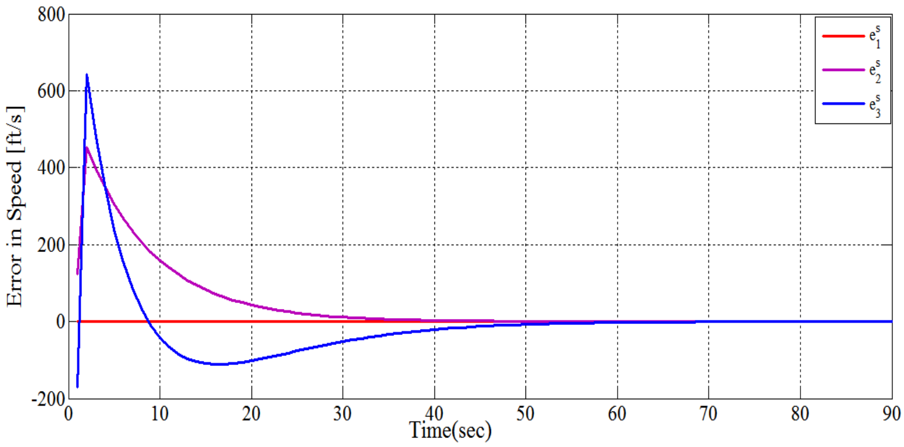


Figure 4.4: Error in speed of vehicles V_1, V_2 and V_2 (Non-iterative)

4.4.2 Iterative Algorithm

The decentralizing state feedback gain by employing iterative algorithm is obtained as

$$K_{Diter} = \begin{bmatrix} -2.0010 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -2.1608 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.0207 & 0 & 2.0113 & 0 & -2.0010 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.3481 & 0 & 1.6071 & 0 & -2.1608 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.0207 & 0 & 2.0113 & 0 & -2.0010 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.3481 & 0 & 1.6071 & 0 & -2.1608 \end{bmatrix}$$

which places closed loop eigen values at the following locations:

4 poles at $-1.0005 \pm 1.0051j$

4 poles at $-1.0804 \pm 0.6632j$

1 pole at -2.0010 and

1 pole at -2.1608

The closed loop eigen values in the complex plane are shown in fig.(4.5). The simulation result is shown in fig.(4.6) for iterative algorithm by considering same set of initial conditions as considered in iterative. Position co-ordinates are in feet.

Horizontal distances between vehicles V_1 and V_2, and V_2 and V_3 for iterative algorithm by considering one set of initial conditions are shown in fig.(4.7). The distances are in feet. In fig(4.8), the errors in speed of vehicles V_1, V_2 and V_3 are shown. speeds are in [ft/s].

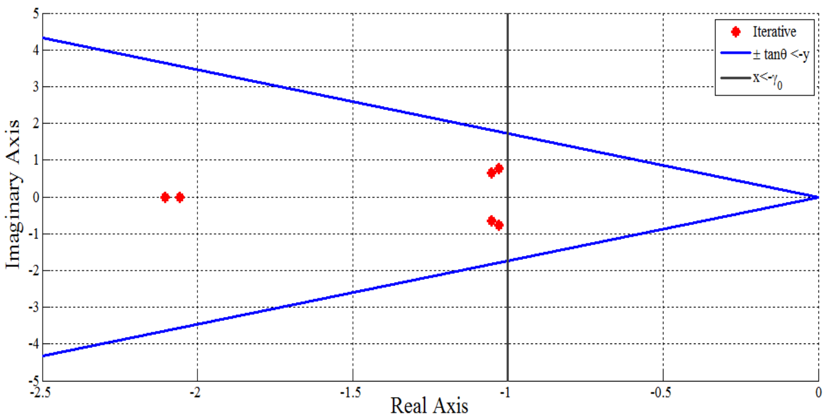


Figure 4.5: Closed loop eigen values (Decentralized Control-Iterative algorithm)

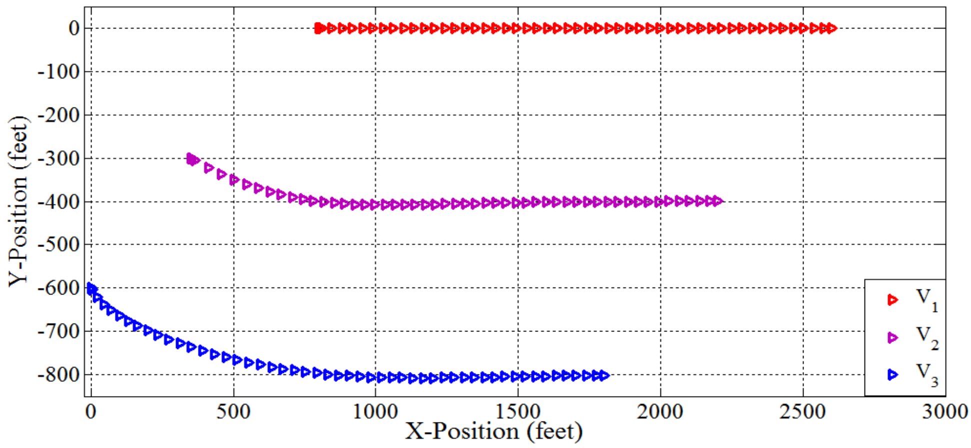


Figure 4.6: Snapshots of the formation for one set of initial conditions - Decentralized (Iterative algorithm)

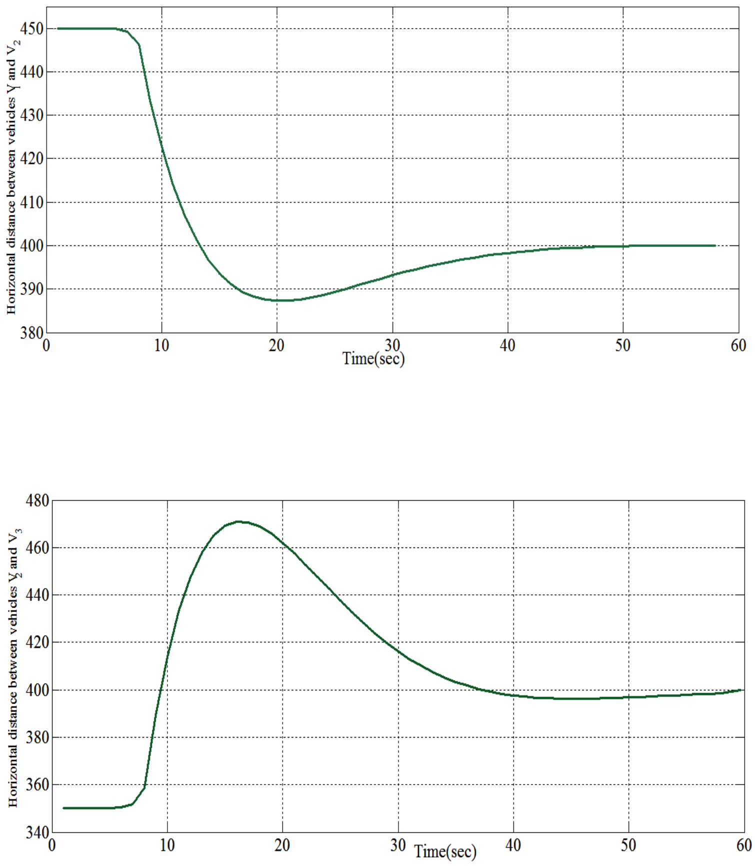


Figure 4.7: Horizontal distances between vehicles V_1 and V_2, and V_2 and V_3 (Iterative algorithm)

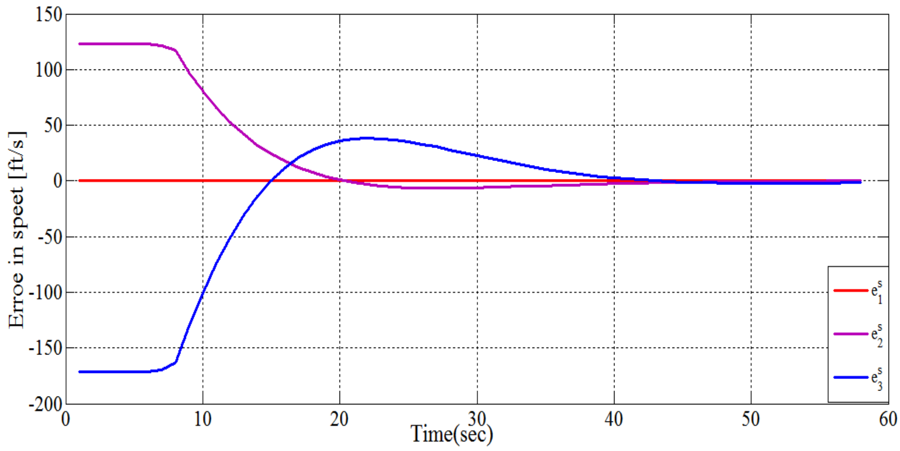


Figure 4.8: Error in speed of vehicles V_1, V_2 and V_2 (Iterative algorithm)

	Non-iterative	**Iterative**
Homotopy method employed for individual subsystem with expansion and contraction of the system	<u>Gain Parameters:</u> $\tilde{K}_1 = 10^8 * \begin{bmatrix} -1.2177 & 0 \\ 0 & -1.1860 \end{bmatrix}$ $\tilde{K}_2 = 10^8 * \begin{bmatrix} -0.3201 & 0 \\ 0 & -0.3117 \end{bmatrix}$ $\tilde{K}_3 = 10^8 * \begin{bmatrix} 3.1243 & 0 \\ 0 & 3.0431 \end{bmatrix}$ <u>Eigen values:</u> 2 at -2.57 1 at -121767751 1 at -118602735 and 1 at -121767754	<u>Gain Parameters:</u> $\tilde{K}_1 = \begin{bmatrix} -2.0010 & 0 \\ 0 & -2.1608 \end{bmatrix}$ $\tilde{K}_2 = \begin{bmatrix} 0.0207 & 0 \\ 0 & 0.3481 \end{bmatrix}$ $\tilde{K}_3 = \begin{bmatrix} 2.0113 & 0 \\ 0 & 1.6071 \end{bmatrix}$ <u>Eigen values:</u> -1.0005 $\pm$ 1.0051j -1.0804 $\pm$ 0.6632j -2.1608 and -2.0010
Homotopy method employed for whole system without expansion and contraction of the system	<u>Gain Parameters:</u> $\tilde{K}_1 = 10^5 * \begin{bmatrix} -0.2136 & 0 \\ 0 & -0.2849 \end{bmatrix}$ $\tilde{K}_2 = 10^5 * \begin{bmatrix} 0.5569 & 0 \\ 0 & 0.7429 \end{bmatrix}$ $\tilde{K}_3 = 10^5 * \begin{bmatrix} 2.1801 & 0 \\ 0 & 2.9081 \end{bmatrix}$ <u>Eigen values:</u> 2 at -21351 2 at -28483 4 at -10.21 1 at -21362 and 1 at -28493	<u>Gain Parameters:</u> $\tilde{K}_1 = \begin{bmatrix} -2.1046 & 0 \\ 0 & -2.0559 \end{bmatrix}$ $\tilde{K}_2 = \begin{bmatrix} 0.0144 & 0 \\ 0 & -0.0000 \end{bmatrix}$ $\tilde{K}_3 = \begin{bmatrix} 1.5471 & 0 \\ 0 & 1.6393 \end{bmatrix}$ <u>Eigen values:</u> -1.0523 $\pm$ 0.6631j -1.0523 $\pm$ 0.6631j -1.0279 $\pm$ 0.7633j -1.0279 $\pm$ 0.7633j -2.1046 and -2.0559
%age difference between the two above results $\frac{\lVert K_{D1}\rVert - \lVert K_{D2}\rVert}{\lVert K_{D1}\rVert} * 100\%$	99.9%	8.22%

Where K_{D1} denotes controller gain obtained for original system when homotopy method employed for single subsystem,

K_{D2} denotes controller gain obtained for original system when homotopy method employed for original system.

4.4.3 Performance of the System

If controller designed for the expanded system $\tilde{S}$ is contractible to controller for the original system S, then the stability of subsystems and its interconnections in the expanded space implies the stability of expanded system which shows the stability of original system.

The stability of system is verified by using standard Lyapunov stability theory described in chapter 2. If controller gain obtained by employing iterative algorithm stabilizes the original system, then it implies the stability of controller obtained by using non-iterative algorithm. The System model having 3 vehicles is

$$\begin{bmatrix} \dot{e}_1^s \\ \dot{e}_2^d \\ \dot{e}_2^s \\ \dot{e}_3^d \\ \dot{e}_3^s \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ I & 0 & -I & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & I & 0 & -I \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} e_1^s \\ e_2^d \\ e_2^s \\ e_3^d \\ e_3^s \end{bmatrix} + \begin{bmatrix} I & 0 & 0 \\ 0 & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & 0 \\ 0 & 0 & I \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$$

The expanded system

$$\begin{bmatrix} \dot{\tilde{e}}_1 \\ \dot{\tilde{e}}_2 \\ \dot{\tilde{e}}_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & I & 0 & -I & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & I & 0 & -I \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \tilde{e}_1 \\ \tilde{e}_2 \\ \tilde{e}_3 \end{bmatrix} + \begin{bmatrix} I & 0 & 0 & 0 & 0 \\ 0 & I & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & I & 0 & 0 \\ 0 & 0 & 0 & I & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & I \end{bmatrix} \begin{bmatrix} \tilde{u}_1 \\ \tilde{u}_2 \\ \tilde{u}_3 \end{bmatrix}$$

and obtained controller gain in the expanded space

$$\tilde{K}_{Diter} = \begin{bmatrix} \tilde{K}_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \tilde{K}_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \tilde{K}_2 & \tilde{K}_3 & \tilde{K}_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \tilde{K}_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & \tilde{K}_2 & \tilde{K}_3 & \tilde{K}_1 \end{bmatrix}$$

Where

$$\tilde{K}_1 = \begin{bmatrix} -2.0010 & 0 \\ 0 & -2.1608 \end{bmatrix}$$

$$\tilde{K}_2 = \begin{bmatrix} 0.0207 & 0 \\ 0 & -0.3481 \end{bmatrix}$$

$$\tilde{K}_3 = \begin{bmatrix} 2.0113 & 0 \\ 0 & 1.6071 \end{bmatrix}$$

The modified controller gain

$$\tilde{K}_{DMiter} = \begin{bmatrix} \tilde{K}_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \tilde{K}_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \tilde{K}_2 & \tilde{K}_3 & \tilde{K}_1 & 0 & 0 & 0 \\ 0 & \tilde{K}_2 & \tilde{K}_3 & 0 & \tilde{K}_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & \tilde{K}_2 & \tilde{K}_3 & \tilde{K}_1 \end{bmatrix}$$

The closed loop system $\tilde{S}_f$ is obtained as

$$\begin{aligned} \tilde{S}_f : \quad \dot{\tilde{x}} &= (\tilde{A} + \tilde{B}\tilde{K}_{DMiter})\tilde{x} \\ &= \tilde{A}_f \tilde{x} \end{aligned}$$

where

$$\tilde{A}_f = \left[\begin{array}{c|ccc|ccc} \tilde{K}_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & \tilde{K}_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & I & 0 & -I & 0 & 0 & 0 \\ 0 & \tilde{K}_2 & \tilde{K}_3 & \tilde{K}_1 & 0 & 0 & 0 \\ \hline 0 & \tilde{K}_2 & \tilde{K}_3 & 0 & \tilde{K}_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & I & 0 & -I \\ 0 & 0 & 0 & 0 & \tilde{K}_2 & \tilde{K}_3 & \tilde{K}_1 \end{array}\right]$$

For stability of S_1

$$P_1 = \begin{bmatrix} 13.14 & 0 \\ 0 & 12.09 \end{bmatrix}$$

Thus,the time-derivative of $\tilde{V}_1(\tilde{x}_1)$ satisfies

$$\begin{aligned}\dot{\tilde{V}}_1(\tilde{x}_1)_{S_1} &= -\tilde{x}_1^T \begin{bmatrix} 52.59 & 0 \\ 0 & 52.35 \end{bmatrix} \tilde{x}_1 \\ &\leq -52.59 \left\|\tilde{x}_1\right\|^2 \end{aligned} \tag{4.27}$$

where $\|\tilde{x}_1\|$ represents the Euclidean norm of $\tilde{x}_1$.The closed loop eigen values are located at -2.1608 and -2.0010.

For stability of S_2 and S_3

$$P_i = \begin{bmatrix} 12.31 & 0 & 9.12 & 0 & -3.7 & 0 \\ 0 & 10.82 & 0 & 8.73 & 0 & -2.75 \\ 9.12 & 0 & 28.32 & 0 & -5.59 & 0 \\ 0 & 8.73 & 0 & 27.92 & 0 & -6.71 \\ -3.70 & 0 & -5.59 & 0 & 9.99 & 0 \\ 0 & -2.75 & 0 & -6.71 & 0 & 9.94 \end{bmatrix}$$

where $\{i = 2, 3\}$ and the time-derivative of $\tilde{V}_i(\tilde{x}_i)$ satisfies

$$\begin{aligned}\dot{\tilde{V}}_i(\tilde{x}_i)_{S_i} &= -\tilde{x}_i^T \begin{bmatrix} 31.18 & 0 & -2.5121 & 0 & -0.32 & 0 \\ 0 & 31.24 & 0 & -2.30 & 0 & 0.09 \\ -2.51 & 0 & 22.53 & 0 & -2.97 & 0 \\ 0 & -2.30 & 0 & 21.57 & 0 & -2.56 \\ -0.32 & 0 & -2.97 & 0 & 28.77 & 0 \\ 0 & 0.09 & 0 & -2.56 & 0 & 29.55 \end{bmatrix} \tilde{x}_i \\ &\leq -31.99 \left\|\tilde{x}_i\right\|^2 \end{aligned} \tag{4.28}$$

The closed loop eigen values are located at $-1.0005 \pm 1.0051j$, $-1.0804 \pm 0.6632j$, -2.0010 and -2.1608.

For the interconnection effects

$$\dot{\tilde{V}}_i(\tilde{x}_i)_{S_j} = 0, \quad (i, j) = (2, 1)(3, 1)(1, 2)(1, 3)(2, 3) \tag{4.29}$$

$$\begin{aligned}\dot{\tilde{V}}_3(\tilde{x}_3)_{S_2} &= 2 * \tilde{x}_3^T \begin{bmatrix} 0.25 & 0 & 24.75 & 0 & 0 & 0 \\ 0 & 3.77 & 0 & 17.39 & 0 & 0 \\ 0.19 & 0 & 18.34 & 0 & 0 & 0 \\ 0 & 3.04 & 0 & 14.03 & 0 & 0 \\ -0.08 & 0 & -7.45 & 0 & 0 & 0 \\ 0 & -0.96 & 0 & -4.42 & 0 & 0 \end{bmatrix} \tilde{x}_2 \\ &\leq 63.39 \left\|\tilde{x}_3\right\| \left\|\tilde{x}_2\right\| \end{aligned} \tag{4.30}$$

For whole system, the time-derivative by using Eqs.(4.29)-(4.32)

$$\dot{\tilde{V}}(\tilde{x})_{\tilde{S}_f} \leq \begin{bmatrix} \|\tilde{x}_1\| \\ \|\tilde{x}_2\| \\ \|\tilde{x}_3\| \end{bmatrix}^T \begin{bmatrix} 52.59 & 0 & 0 \\ 0 & 31.99 & 0 \\ 0 & -63.39 & 31.99 \end{bmatrix} \begin{bmatrix} \|\tilde{x}_1\| \\ \|\tilde{x}_2\| \\ \|\tilde{x}_3\| \end{bmatrix} \tag{4.31}$$

Thus, the stability of the expanded system is established by the positive definiteness of the matrix of right hand side of inequality.

Chapter 5

Discussion and Conclusion

Chapter 5

Discussion and Conclusion

In this thesis, algorithms for designing centralized and decentralized control laws with the aid of pole placement have been discussed. The presented controller places closed loop eigen values near the boundary region with low gain.

Basically, two cases have been discussed. In first case methods for designing centralized and decentralized controller with desired transient performance have been presented for interconnected power system based on pole placement and Lyapunov stability theory. The formulated optimization problems are solved by using non-iterative and iterative algorithm. Then the comparative studies of both algorithms have been done by simulating two-area power system model.

In second case a method for designing decentralized controller with desired transient performance for formation control of a group of unmanned aerial vehicles has been presented based on pole placement and Lyapunov stability theory. The formation is considered as an interconnected subsystem. The original system is expanded into a higher dimensional space where subsystems are appeared as disjoint. In the expanded space static feedback law with pole placement constraints is designed for each subsystem and then contracted back for original system. The optimization problem is solved by using homotopy method for the whole system and then an iterative algorithm is employed to obtain a low gain controller.

We observe the following points:

- The closed loop eigen values are more near to the boundary region in iterative method.
- According to stability criteria, if the poles are shifted towards imaginary axis it reduces the stability of system. Thus the closed loop system becomes less stable in case iterative algorithm as compared to non-iterative. On the other hand, simultaneously iterative algorithm results reduction in controller gain which increases stability. Thus ILMI algorithm provides enough minimum stability, which a closed loop system requires.
- Comparison of Non-iterative and Iterative algorithm

Comparison Points	**Non-iterative algorithm**	**Iterative algorithm**
Controller gain	large	less
Location of poles	far away from boundary	near the boundary
Computation time	less	more
Disturbance reduction	more	less
Stability	more	less
Establishment Cost	high	less

5.1 Scope for Future Work

- In the future the application of dynamic controller can be considered.
- The information structure constraints with different overlapping structure can be considered for formation control.
- Discrete time domain will be considered for control tasks in future research.

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